Unity Root Matrix Theory. Books 1-5 Aug 2019 Markup

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This document provides corrections to all five currently published Books.

Only algebraic errata or incorrect statements are documented herein. Otherwise, grammar and typographical errors are not corrected except when they affect the understanding of the content.

This document is a working document and is updated as and when new mistakes are detected.

Corrections are not provided for the online free papers, albeit such corrections are made directly on occasion, as and when.

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Table of Contents

1 Book 1 Physics in Integers

Paper 1 Unity Root Matrix Theory

pg. 11

Paragraph after equation (4.6) beginning,

Lastly, it is noted...

This paragraph is incorrect and matrix **A** is not a projection operator, i.e. it does not project an arbitrary vector on to eigenvector **X**. The usage of symbol **X** here for a vector makes it look like an arbitrary vector, but it is an eigenvector of **A** and therefore, by definition, multiplying **X** by **A** will always return an eigenvalue multiple of **X**, which is not the case for an arbitrary vector. In fact, projection operators do enter URMT later in Section 4 of Book 4.

pg. 43

The equations at the foot of the page have an extraneous '$\delta\varepsilon$' symbol in front of them, and their correct layout is as follows, with the '$\delta\varepsilon$' removed from all bar the first line

$\delta\varepsilon$ terms

(A1.5a) $0 = -x^2 + xyR + xz\overline{Q}$

(4.1a) $x = Ry + \overline{Q}z$.

$\eta\varepsilon$ terms

(A1.5b) $0 = -y^2 + yzP + xy\overline{R}$

(4.1b) $y = \overline{R}x + Pz$.

$\eta\delta$ terms

(A1.5c) $0 = -z^2 + xzQ + yz\overline{P}$

(4.1c) $z = Qx + \overline{P}y$.

Paper 2 Pythagorean Triples as Eigenvectors...

pg. 72

Paragraph after equation (9.16), reference [3] should read reference [4].

pg. 91

Equations (9.9) to (9.11) are missing a transpose symbol on the right-hand side, and should read as follows:

(9.9)
$$\mathbf{X}^{+} \wedge \mathbf{X}^{0} = -\mathbf{X}^{0} \wedge \mathbf{X}^{+} = \left(\mathbf{X}_{-}\right)^{T}$$
$$\mathbf{X}_{+} \wedge \mathbf{X}_{0} = -\mathbf{X}_{0} \wedge \mathbf{X}_{+} = \left(\mathbf{X}^{-}\right)^{T}$$

(9.10)
$$\mathbf{X}^{-} \wedge \mathbf{X}^{+} = -\mathbf{X}^{+} \wedge \mathbf{X}^{-} = 2\left(\mathbf{X}_{0}\right)^{T}$$
$$\mathbf{X}_{-} \wedge \mathbf{X}_{+} = -\mathbf{X}_{+} \wedge \mathbf{X}_{-} = 2\left(\mathbf{X}^{0}\right)^{T}$$

(9.11)
$$\mathbf{X}^{0} \wedge \mathbf{X}^{-} = -\mathbf{X}^{-} \wedge \mathbf{X}^{0} = \left(\mathbf{X}_{+}\right)^{T}$$
$$\mathbf{X}_{0} \wedge \mathbf{X}_{-} = -\mathbf{X}_{-} \wedge \mathbf{X}_{0} = \left(\mathbf{X}^{+}\right)^{T} .$$

Paper 3 Geometric and Physical Aspects

pg. 123

Second to last line, the text

radius in the $x-y$ *plane*

should read

radius in the $P-Q$ *plane.*

pg. 128

Line before equation (6.9), the subscript 'm0' on $\delta\mathbf{X}_{m0}$ should be 'm-':

before

$\delta\mathbf{X}_{m0}$

correction

$\delta\mathbf{X}_{m-}$

pg. 150

Equation (12.13)

before

$m >> 0 , \delta m = 1$

correction

$|m| >> 0 , |\delta m| = 1$

Paper 4 Solving Unity Root Matrix Theory

pg. 217

Second paragraph, 7th line, text *it would be imply* should read *it would imply*.

pg.219

Paragraph before equation (5.8)

Before

$x^n = -y^n \bmod z$

Corrected

$x^n \equiv -y^n \pmod z$

pg. 242

Equation (B2.1), extraneous superscript

Before

$U_7^3 = \{1,2,4\}$.

corrected

$U_7 = \{1,2,4\}$

The superscript ‘3’ was originally used to denote the exponent to which the root pertains, i.e. cubic here, but this superscript notation was dispensed with.

pg. 243

Equation (B2.3), see also above,

Before

(B2.3) $U_{13}^3 = \{1,3,9\}$

corrected

$U_{13} = \{1,3,9\}$

Paper 5 Unifying Concepts

[1]#5 Unifying concepts

pg. 305

Last paragraph, third line from bottom, the text $\mathbf{X}^0 = (\overline{P}, \overline{Q}, \overline{R})$ should read $\mathbf{X}^0 = (\overline{P}, -\overline{Q}, \overline{R})$

pg. 342

last equation from bottom, un-numbered,

before

$\alpha,\beta,\gamma \to C^2\alpha C^2,\beta C^2,\gamma C^2$

corrected

$\alpha,\beta,\gamma \to \alpha C^2,\beta C^2,\gamma C^2$

pg. 360

Equation (B7), the sign of k should be positive, i.e.

before

$0 = 7^3 + 13^3 - 635^3 - k.7.13.635$

correction

$0 = 7^3 + 13^3 - 635^3 + k.7.13.635$

Paper 6 Non-Unity Eigenvalues

pg. 327

Paragraph after equation (4.2f), first line, the text

the residue is now $\pm C^2$

should read

the residue is now $\pm C^n$

2 Book 2 Higher Dimensional Extensions

pg. 54

Equations (3.90) and (3.91) are both missing the scalar factor s as in before

$$\mathbf{X}^0 = \begin{pmatrix} 0 \\ \mathbf{X}_3 \end{pmatrix}$$

corrected

$$\mathbf{X}^0 = \begin{pmatrix} 0 \\ s\mathbf{X}_3 \end{pmatrix}$$

In addition equation (3.91) should read, in full,

$$\mathbf{X}^0 = \begin{pmatrix} 0 \\ s\mathbf{X}_3 \end{pmatrix} = s\mathbf{X}^0\mathbf{X}_3 = 0$$

For information, the four-element, column vector **X,** referred to in the paragraph above equation (3.19), is first defined in (2.4) p20 but without the scalar factor *s*.

pg. 175

Second to last paragraph, first line, the text *Pythagorean triple* should read *Pythagorean Quadruple*

pg. 319

Equation (E6), the max range of the zero eigenvector index is $n-3$,

before

$j = 0 \ldots n-1$

corrected

$j = 0 \ldots n-3$

pg. 339

Definition (I4) An **Excess Dimension**

before

$j = 3 < r < n$

corrected

$j = 3 < r \leq n$

3 Book 3 Mathematical and Physical Advances Volume I

pg. 13

First paragraph, second to last line, the text

may be in reals or complex

should read

may be in real or complex numbers

pg. 21

Equations (1.54) and (1.55) omit transposition of vector $\mathbf{X}$

before

(1.54) $\mathbf{X}^- = (\mathbf{X} \quad -|\mathbf{X}|)$

(1.55) $\mathbf{X}^+ = (\mathbf{X} \quad |\mathbf{X}|).$

correction

(1.54) $\mathbf{X}^- = (\mathbf{X}^T \quad -|\mathbf{X}|)$

(1.55) $\mathbf{X}^+ = (\mathbf{X}^T \quad |\mathbf{X}|).$

pg. 33

The text

Since $v = c$ (1.108) this is all quite basic as it means that $\mathbf{A}$ is really just the identity matrix i.e.

should read

Since $v = c$ *(1.108) this reduces* $\mathbf{A}$ *to the following simplistic form:*

pg. 62

Equation (2.111d), $\mathbf{X}_-$ *should be* $\mathbf{X}_{i-}$ *as in*

$$\mathbf{X}^{i+} = \left(\mathbf{TX}_{i-}\right)^T$$

pg. 126

Last paragraph, first line, the text

In the URM3 SPI, Appendix (J), $\mathbf{X}_0$ *and* $\mathbf{X}_-$ *are treated as velocity and acceleration vectors*

should read

In the dual SPI (4.104), $\mathbf{X}_0$ *and* $\mathbf{X}_-$ *are treated as velocity and acceleration vectors*

pg. 143

Second paragraph, second line from end, equation

$$E^2 = (pc)^2 + E_0^2$$

shoud read

$$E^2 = \left(pc\right)^2 + E_0^2$$

pg. 329

Paragraph before Section (10-7), first line, the extraneous superscript of ‘2’ on the equation for ***r*** should be removed.

before

$$r \neq \sqrt{x^2 + y^2 + z^2}^{\,2}$$

correction

$$r \neq \sqrt{x^2 + y^2 + z^2}$$

pg. 330

First equation, top of page, un-numbered, the Pythagorean **T** operator (see also Appendix (A34)) is not equal to the negative inverse.

before

$$\mathbf{T} = \mathbf{T}^T = \mathbf{T}^{-1} = -\mathbf{I}_3 = \begin{pmatrix} +1 & 0 & 0 \\ 0 & +1 & 0 \\ 0 & 0 & -1 \end{pmatrix}$$

correction

$$\mathbf{T} = \mathbf{T}^T = \mathbf{T}^{-1} = \begin{pmatrix} +1 & 0 & 0 \\ 0 & +1 & 0 \\ 0 & 0 & -1 \end{pmatrix}$$

pg. 330

Equation (10.81), the Skew T operator should be equal to the negative of the identity matrix

before

$$\mathbf{T} = \mathbf{T}^T = \mathbf{T}^{-1} = \begin{pmatrix} +1 & 0 & 0 \\ 0 & +1 & 0 \\ 0 & 0 & -1 \end{pmatrix}$$

correction

$$\mathbf{T} = \mathbf{T}^T = \mathbf{T}^{-1} = -\mathbf{I}_3 = \begin{pmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix}$$

pg. 331

Second paragraph, third line, the text '*they form an orthogonal set*' is not true and they do not form an orthogonal set. Whilst the equations (10.87) remain correct, they do not cover the following two inner products, that are non-zero, and hence $\mathbf{X}_{i+}$ is not orthogonal to $\mathbf{X}_{i-}$ and vice-versa:

$$\mathbf{X}_{i+} \cdot \mathbf{X}_{i-} = -2C^2$$

$$\mathbf{X}_{i-} \cdot \mathbf{X}_{i+} = -2C^2$$

Hence the set { $\mathbf{X}_{i+}, \mathbf{X}_{i0}, \mathbf{X}_{i-}$ } is not truly orthogonal.

pg. 467

Equation (G3) subscript '+' should be '3+'

Before

$$\frac{d\mathbf{X}_0}{dt_3} = -\mathbf{X}_+$$

correction

$$\frac{d\mathbf{X}_0}{dt_3} = -\mathbf{X}_{3+}$$

4 Book 4 Mathematical and Physical Advances Volume II

pg. 115

Section (4-4)

The outer product definition in Section (4-4) is confusing at best, and wrong in some contexts in which it is used, particularly with what follows it on Projection Operators.

The loosest and most general definition of the outer vector product, at least within the context of URMT, is that of the product of a column vector with a row vector to give a matrix result.

With such a loose definition then, if $\mathbf{X}$ and $\mathbf{Y}$ are two column vectors, the definition (4.80) given in Section (4-4), i.e. $\mathbf{A}_{ij} = \mathbf{X}_i \left(\mathbf{Y}^T\right)_j$ would be correct since the transpose of **Y**, i.e. $\mathbf{Y}^T$, would be a row vector, and so the result is the matrix **A**.

However, more generally outside of URMT, if **Y** were complex, which the definition states it can be, then the transpose complex conjugate would be more usual, i.e. $\mathbf{A}_{ij} = \mathbf{X}_i \left(\mathbf{Y}^T\right)^*{}_j$. Nevertheless, use of the transpose conjugate is not usually required in URMT by virtue that the definition of a URMT conjugate vector is explicity defined by the **T** operator relations (See Appendix(E)) and implictly contains conjugation when acting on URMT eigenvectors. To avoid any confusion, it is safest to limit the strict definition to URMT usage as follows with the below corrected definition.

(4-4) Definition: Outer Product in the context of URMT
The outer product of two vectors, within the context of URMT, is the product of a column eigenvector $\mathbf{X}$ with a reciprocal or conjugate, row eigenvector $\mathbf{Y}$, both of which may be complex, to give a square matrix result, where the matrix element of the ith row and jth column, denoted by $\mathbf{A}_{ij}$ (or $\mathbf{A}(i, j)$ in some cases herein), is given by

(4.80) $\mathbf{A}_{ij} = \mathbf{X}_i \mathbf{Y}^j$, $i, j = 1..n$, $\mathbf{X}, \mathbf{Y} \in \mathbb{C}^{\mathrm{n}}$.

$$\mathbf{X} \in \{\mathbf{X}_+, \mathbf{X}_0, \mathbf{X}_-\}$$
$$\mathbf{Y} \in \{\mathbf{Y}^+, \mathbf{Y}^0, \mathbf{Y}^-\}$$

A relaxation to the looser definition...

Outside of the above strict definition, with its restriction to URMT eigenvectors, the outer product definition may be applied more loosely within the context of URMT, to arbitrary column vectors **X** and row vectors **Y**. In such a case, the vector **Y** may well be formed from the transpose or transpose conjugate of a column vector from the set of **X** but it needn't be. And finally, keeping with this relaxation, then if both **X** and **Y** are column vectors (**Y** no longer a row vector), then the original outer product definition $\mathbf{A}_{ij} = \mathbf{X}_i \left(\mathbf{Y}^T\right)_j$, using just the transpose to convert **Y** from a column to a row vector, or the transpose conjugate form $\mathbf{A}_{ij} = \mathbf{X}_i \left(\mathbf{Y}^T\right)^*{}_j$ remains valid.

Following the definition (4.80) are some 'Notes' that also now require correction, as give below.

Notes (corrected)
Unlike the earlier statements on vectors $\mathbf{X}$ and $\mathbf{Y}$ in AVE I and II, which are stated as real (2.2), this definition includes complex because, the vectors $\mathbf{X}$ and $\mathbf{Y}$ are usually URMT eigenvectors.

The outer vector product is not, in general, commutative, i.e. $\mathbf{X}_i\mathbf{Y}^j$ is not generally equal to $\mathbf{Y}_i\mathbf{X}^j$, except along the lead diagonal.

As regards notation, some texts explicitly use the symbol $\otimes$ for the outer product, i.e.

(4.86) $\mathbf{A} = \mathbf{X} \otimes \mathbf{Y}$.

However, in URMT, the symbol is not really required since the laws of vector (matrix) algebra give an unambiguous, matrix result, i.e. the product of an $n \times 1$ column vector multiplied by a $1 \times n$ row vector (the transposed form of a column vector) gives an $n \times n$ matrix result. Compare this with the inner vector (dot) product defined as $\mathbf{Y} \cdot \mathbf{X} = \sum_i \mathbf{Y}^i \mathbf{X}_i$, and is the product of a $1 \times n$ row vector ($\mathbf{Y}^T$) by an $n \times 1$ column vector $\mathbf{X}$ to give a 1×1, scalar result. In other words, the outer product of two vectors gives a matrix, and the inner product gives a single number. Note that the summation symbol on the inner product is dropped as assumed implicit over the repeated index *i*, known in Tensor Calculus as the 'Einstein Summation Convention', i.e. $\mathbf{Y}^i \mathbf{X}_i$ is used instead of $\sum_i \mathbf{Y}^i \mathbf{X}_i$.

pg. 249

Equation (10.24) should read as follows, where the subscript on $\mathbf{X}_{4i+}$ has been changed to $\mathbf{X}_{4i\pm}$, and likewise for $\mathbf{X}_{4j+}$:

(10.24)

$$\mathbf{Q}_{\underline{\mathbf{q}}}\mathbf{X}_{4i\pm} = \pm i|\underline{\mathbf{q}}|\mathbf{X}_{4i\pm}$$

$$\mathbf{Q}_{\underline{\mathbf{q}}}\mathbf{X}_{4j\pm} = \pm i|\underline{\mathbf{q}}|\mathbf{X}_{4j\pm}$$

On the same page, 249, second line before equation (10.26), the equation $\lambda_{\underline{\mathbf{q}}} \pm i|\underline{\mathbf{q}}|$ is missing the equality symbol and should read $\lambda_{\underline{\mathbf{q}}} = \pm i|\underline{\mathbf{q}}|$.

pg. 363

Equation (15.72) $\mathbf{X}_{4\pm}$ should be $\mathbf{X}_{4i\pm}$ as per (15.68a).

Appendix (I) pg. 482

Definition of the exterior product.

Similar comments apply to this definition as per the errata, further above, for the Outer Product, pg. 115, Section (4-4). The definition given is defined in terms of column vectors **X** and **Y** and their transpose as in the below extract.

(I40) The **Exterior Product** (symbol) of two, arbitrary, n-dimensional vectors is defined in URMT as

$\mathbf{X} \wedge \mathbf{Y} = \mathbf{X}\mathbf{Y}^T - \mathbf{Y}\mathbf{X}^T = \mathbf{X} \otimes \mathbf{Y} - \mathbf{Y} \otimes \mathbf{X}$, the exterior product of **X** and **Y**,

which is the difference of the two outer vector products $\mathbf{X}\mathbf{Y}^T$ and $\mathbf{Y}\mathbf{X}^T$ (or $\mathbf{X} \otimes \mathbf{Y}$ and $\mathbf{Y} \otimes \mathbf{X}$).

This is a loose definition and, exactly as per the errata highlighted for the outer product, further above, the transpose could also be the transpose complex conjugate.

A stricter definition is therefore as follows:

The exterior product of two vectors, within the context of URMT, is the difference in the product of the column eigenvector $\mathbf{X}$ with the reciprocal or conjugate, row eigenvector $\mathbf{Y}$, and the product of the column eigenvector **Y** with the reciprocal or conjugate, row eigenvector **X**, where both vectors may be complex. The result of the product difference is an anti-symmetric square matrix, where the matrix element of the ith row and jth column, denoted by $\mathbf{A}_{ij}$ (or $\mathbf{A}(i, j)$ in some cases herein), is given by

(4.80) $\mathbf{A}_{ij} = \mathbf{X}_i \mathbf{Y}^j - \mathbf{X}_j \mathbf{Y}^i,\ i, j = 1..n,\ \mathbf{X}, \mathbf{Y} \in \mathbb{C}^n.$

$$\mathbf{X} \in \{\mathbf{X}_+, \mathbf{X}_0, \mathbf{X}_-\}$$
$$\mathbf{Y} \in \{\mathbf{Y}^+, \mathbf{Y}^0, \mathbf{Y}^-\}$$

This is actually the definition as used in the main body of the text in Part III on the Exterior Product Formulation and does not conflict with the text.

It is possible that the looser definition, as given by (I40), applies in some cases.

Appendix (L) pg. 510

Definition (L6d) of the outer product. See the comment for Section (4-4) p115 in this Book 4.

5 Book 5 A Quark Flavour Model

Algebraic Corrections

pg. 20

Equation (1.53), last of the three equations, $\mathbf{J}_0$ should be $\mathbf{J}_x$.

before

$[\mathbf{J}_+, \mathbf{J}_-] = 2\hbar\mathbf{J}_0$

correction

$[\mathbf{J}_+, \mathbf{J}_-] = 2\hbar\mathbf{J}_x$

pg. 25

Second to last paragraph, second line, the text

y and z-axis

should read

y and z axes

pg. 88

Equation (4.13c) is missing an equality sign

before

(4.31c) $\bar{\mathbf{u}}\bar{\mathbf{I}}_+ = \bar{\mathbf{u}}\left(-\mathbf{u}\bar{\mathbf{d}}\right) - \left(\bar{\mathbf{u}}\mathbf{u}\right)\bar{\mathbf{d}} = -\bar{\mathbf{d}}$ using $\bar{\mathbf{u}}\mathbf{u} = 1$ (4.17a)

correction

(4.31c) $\bar{\mathbf{u}}\bar{\mathbf{I}}_+ = \bar{\mathbf{u}}\left(-\mathbf{u}\bar{\mathbf{d}}\right) = -\left(\bar{\mathbf{u}}\mathbf{u}\right)\bar{\mathbf{d}} = -\bar{\mathbf{d}}$ using $\bar{\mathbf{u}}\mathbf{u} = 1$ (4.17a)

pg. 197

First paragraph, last sentence

In fact, every one of the seven matrices $\boldsymbol{\lambda}_i$, $i = 1...7$ *squares to the identity, with a full, cubic, characteristic equation [5] given by*

This is wrong since they do not square exactly to the identity and would be better worded as follows:

In fact, every one of the seven matrices $\boldsymbol{\lambda}_i$, $i = 1...7$ *squares to a diagonal matrix with unity occupying two of the three diagonal positions, i.e. almost like the identity but with one of the diagonal positions zero. Each matrix has a full, cubic, characteristic equation [5] given by*

pg. 202

Top of page, equations (11.17), the identity equation for **I** (last equation) is actually only true for the 3x3 Gell-Mann matrices, as covered by URM3, and not URM6 dealt with later in Chapter 13 onward. Whilst this point may be clear given that the entire chapter 11 is titled 'Gell-Mann Matrices', the equations (11.18) are supposed to be valid for SU(N), N>=3, i.e currently URM3 to URM6.

pg. 203

Top of page, equations (11.18) continued from page 202, the second equation involving the identity, i.e. the equation for $\mathbf{X}_{S0}\mathbf{X}^{S0}$, is actually only true for the 3x3 Gell-Mann matrices, see the above correction to page 202 for more details.

pg. 268

The last expression for $\mathbf{X}_{6+}$ in equation (13.24) uses the wrong initial vector $\mathbf{X}'_{6-}$ instead of the correct $\mathbf{X}'_{6+}$ and should read

$$\mathbf{X}_{6+} = -(f_3 + f_4 + f_5 + f_6)^2 \mathbf{X}_{6-} + 2 f_3 \mathbf{X}'_{60A} + 2 f_4 \mathbf{X}'_{60B}$$
$$+ 2 f_5 \mathbf{X}'_{60C} + 2 f_6 \mathbf{X}'_{60D} + \mathbf{X}'_{6+}$$

pg. 287

Equation (13.80). The bottom right matrix element '$\mathbf{0}^3$' of matrix $\mathbf{I}_3$ should really be written as a 3x3 zero matrix, i.e. $\mathbf{0}_3^3 = \mathbf{0}_3 \otimes \mathbf{0}^3$. Note too that the subscript on matrix symbol $\mathbf{I}_3$ is not the same as the numeric subscript/superscript 3 in $\mathbf{0}_3^3$. $\mathbf{I}_3$ is just a name for the isospin operator whereas subscript/superscript 3 denotes the number of elements in the vector. Neither is $\mathbf{I}_3$ an identity matrix as its use of capital '**I**' might suggest.

Appendix (D) pg. 354

Second paragraph, first sentence starting with

The N matrix elements of a Lie group are actually derivable from the $N^2 - 1$ *generator matrices plus the identity matrix*

should read

The matrix elements of a Lie group are actually derivable from the $N^2 - 1$ *generator matrices plus the identity matrix*

And on the the same page, last paragraph starting with

Now, for a Lie group, if $\mathbf{G}_k$ *is a generator,* $k = 1 \ldots N - 1$

should read

Now, for a Lie group, if $\mathbf{G}_k$ *is a generator,* $k = 1 \ldots N^2 - 1$

Appendix (D) pg. 362

Third paragraph, the sentence starting with

The $N-1$ *diagonal generators commute with all other generators,*

and should read

The $N-1$ *diagonal generators commute with each other,*

Appendix (I) pg. 400

Definition of the exterior product (I40).

See the Errata given earlier herein for Book 4, Appendix (I40), pg. 482

Appendix (I) pg. 406

Line after (I39d), the text

see the [4],1.73

should read

see [4],173

Appendix (L) pg. 462

Definition (L6d) of the outer product. See the comment for Section (4-4) pg. 115 in Book 4.