Unity Root Matrix Theory
Solutions to the Coordinate Equation

$$0 = x^n + y^n - z^n + kxyz$$

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Abstract

This document provides integer solutions to the following equation, known as the 'coordinate equation' in Unity Root Matrix Theory (URMT):

$$0 = x^n + y^n - z^n + kxyz$$

An overview of the theory behind the solutions is also provided, plus algorithms to obtain the solutions in full, together with some discussion.

The solutions presented herein are for positive integers (x, y, z), $2 \le x < y < z$, prime, integer exponent $n \ge 3$, and for some integer k, which is non-zero in accordance with the proof of Fermat's Last Theorem (Wiles 1995).

This document is currently intended as a working document, and will be updated with new roots and pertinent information as and when it becomes available. Given this is a relatively early draft it is also likely corrections will be made quickly upon discovery.

Acronyms Used

DCE Dynamical Conservation Equation
FLT Fermat's Last Theorem
gcd greatest common divisor
LDE Linear Diophantine Equation
MFLT Modified FLT
TBC To Be Confirmed
TBD To Be Defined
URMT Unity Root Matrix Theory

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1 Introduction

Unity Root Matrix Theory's origins go right back to [1] and the solution, in integers, of the following Diophantine equation:

(1.0) $0 = x^n + y^n - z^n + kxyz$, the coordinate equation,

subject to the conditions

(1.1a) $x, y, z, n, k \in \mathbb{Z}$, $2 \le x < y < z$, $n \ge 3$, $k \ne 0$,
(1.1b) $\gcd(x, y) = 1$, $\gcd(y, z) = 1$, $\gcd(z, x) = 1$.

See also [2] for some free information on this equation and general URMT – more specifics are given further below in Section (2).

Equation (1.0) is known as the 'coordinate equation' in URMT because its solutions, in integers, are given by the ordered triple (x, y, z) , i.e. coordinates *x*, *y* and *z*, which are elements of an eigenvector to a unity root matrix, detailed further below (2.3).

The coordinate equation is purposefully defined as a modified form of the equation used in Fermat's Last Theorem (FLT), with an added '*k* term' ($kxyz$) to ensure an infinite set of integer solutions for arbitrary exponent *n* (1.1a). It is the integer constant *k*, in the *k* term, that is of key interest in all solutions, and its value is determined from the solution in *x*, *y* and *z*, by straightforward rearrangement of (1.0) to give

(1.2) $k = \dfrac{\left(z^n - x^n - y^n\right)}{xyz}$.

The restrictions given by (1.1a) are those of FLT, but not generally necessary for (1.0), and the equation does have solutions for negative integers and/or a unity value for *x*, i.e. $x = 1$. The coordinate equation does, of course, also have an infinite set of solutions for the quadratic exponent $n = 2$, where *k* is zero for Pythagorean triples, but non-zero otherwise. Furthermore, the value of *z* need not be restricted to greater than *y* or *x* as in (1.1a), and can also be any arbitrary integer. Consequently, some simple or interesting solutions are given outside of the scope of the restrictions (1.1a) but, nevertheless, the bulk of the data is subject to (1.1a). Suffice to note, co-primality in pairs, condition (1.1b), is always adhered to.

Primality of the exponent *n* is also an extra condition, not specified in (1.1), and imposed in accordance with any proof of FLT – proof of the composite, even exponent $n = 4$ is also required. Solutions for prime exponents also form the bulk of solutions for (1.0) but a few even exponent solutions, $n = 4$, are given in Section (9.2).

The coordinate equation is actually a variant of the modified FLT equation (MFLT), first introduced in [3], with a similar k term, notably $kx^n y^n$ instead of $kxyz$ in (1.0). This latter term is simpler to work with since it no longer involves the exponent and, additionally, it makes the entire equation symmetric upon interchange of the coordinates, barring sign when interchanging x and z or y and z. It is this simplification that really started URMT and, whilst MFLT also has a semi-analytic solution, and gives some interesting solutions, plus a conjecture (still unproven after over 10 years, see [4]), it still proved intractable – it was hoped that the MFLT conjecture would be relatively easy to prove – wishful thinking!

2 Theory

The full theory behind the solution to (1.0) and unity roots is given in [1]#1 and [1]#4. The latter also provides the algorithm used to solve for a complete solution, see also Section (5.2) further below.

Three unity roots P, Q, R are defined in terms of x, y, z in (1.0) as follows:

(2.0)

$$P^n \equiv 1 \pmod{x}$$

$$Q^n \equiv 1 \pmod{y}$$

$$R^n \equiv -1 \pmod{z},$$

and so too are three 'conjugate' unity roots $\overline{P}, \overline{Q}, \overline{R}$

(2.1)

$$\overline{P}^n \equiv 1 \pmod{x}$$

$$\overline{Q}^n \equiv 1 \pmod{y}$$

$$\overline{R}^n \equiv -1 \pmod{z}$$

$$P, Q, R \in \mathbb{Z},\ (P, Q, R) \neq (0,0,0).$$

The conjugates $\overline{P}, \overline{Q}, \overline{R}$ relate to their 'standard' forms P, Q, R by the following 'conjugate relations':

(2.2)

$$\overline{P} \equiv P^{n-1} \pmod{x},\ P \equiv \overline{P}^{n-1} \pmod{x}$$

$$\overline{Q} \equiv Q^{n-1} \pmod{y},\ Q \equiv \overline{Q}^{n-1} \pmod{y}$$

$$\overline{R} \equiv R^{n-1} \pmod{z},\ R \equiv -\overline{R}^{n-1} \pmod{z}$$

$$\overline{P}, \overline{Q}, \overline{R} \in \mathbb{Z},\ (\overline{P}, \overline{Q}, \overline{R}) \neq (0,0,0),$$

These six, unity roots populate the unity root matrix **A**, defined as

(2.3) $\mathbf{A} = \begin{pmatrix} 0 & R & \overline{Q} \\ \overline{R} & 0 & P \\ Q & \overline{P} & 0 \end{pmatrix}$.

Defining the column vector $\mathbf{X}_+$ in terms of x, y, z by

(2.4) $\mathbf{X}_+ = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$,

then $\mathbf{X}_+$ is an eigenvector of **A** for unity eigenvalue, i.e.

(2.5a) $\mathbf{AX}_+ = \mathbf{X}_+$.

Note that [2] talks in terms of a more general, non-unity eigenvalue, 'big *C*', $C \in \mathbb{Z}$, $C \geq 1$ but, as explained in [1]#6, the solution can be transformed back to a unity eigenvalue problem, and so a non-unity value of *C* is not used herein.

The three separate equations, represented by (2.5a) are known as the 'dynamical equations' in URMT, and are as follows:

(2.5b) $x = Ry + \overline{Q}z$

(2.5c) $y = \overline{R}x + Pz$

(2.5d) $z = Qx + \overline{P}y$.

Defining a 'Kinetic' term K and 'Potential' term V as

(2.6) $K = P\overline{P} + Q\overline{Q} + R\overline{R}$, $V = PQR + \overline{P}\overline{Q}\overline{R}$,

then the characteristic equation for **A** (2.3), general eigenvalue λ, is

(2.7) $\lambda^3 - K\lambda - V = 0$, see also [1]#6,

and for the specific, unity eigenvalue $\lambda = 1$ this reduces to

(2.8) $1 = K + V$,

expanded in full as

(2.9) $1 = \left(P\overline{P} + Q\overline{Q} + R\overline{R}\right) + \left(PQR + \overline{P}\overline{Q}\overline{R}\right)$.

This equation is known as the 'Dynamical Conservation Equation' in URMT, or 'the DCE' for short. Like the coordinate equation, it is an invariant of the theory. This is because the unity roots are specified as congruences, (2.0) and (2.1), in terms of the coordinates x, y, z, and thus they are not unique. There is, in fact, an infinite set of unity roots for each and every solution in x, y, z but, regardless of which unity roots are used, the DCE is always satisfied.

The congruences, (2.0) and (2.1), can be expanded and re-expressed in terms of x, y, z and three 'dual' variables α, β, γ (they are the duals of x, y, z, see [1]#5 or [2],3) , also known as 'scale' or divisibility' factors, as follows:

(2.10)

$$(1 - P\overline{P}) = \alpha x$$

$$(1 - Q\overline{Q}) = \beta y$$

$$(1 - R\overline{R}) = \gamma z$$

$$\alpha, \beta, \gamma \in \mathbb{Z},\ (\alpha, \beta, \gamma) \neq (0,0,0).$$

By defining a reciprocal, row vector $\mathbf{X}^+$ in terms of α, β, γ by

(2.11) $\mathbf{X}^+ = \begin{pmatrix} \alpha & \beta & \gamma \end{pmatrix}$,

then $\mathbf{X}^+$ is a row eigenvector of to the same, unity eigenvalue as per $\mathbf{X}_+$ (2.5a), i.e.

(2.12) $\mathbf{X}^+\mathbf{A} = \mathbf{X}^+$.

Summing the three equations (2.6) gives

(2.13) $3 - (P\overline{P} + Q\overline{Q} + R\overline{R}) = \alpha x + \beta y + \gamma z$,

and substituting for the kinetic term *K* from (2.6) gives

(2.14) $3 - K = \alpha x + \beta y + \gamma z$.

Using the DCE, to substitute for *K* in terms of the Potential *V*, this becomes the third, key equation in URMT, known as the 'Potential equation'

(2.15) $2 + V = \alpha x + \beta y + \gamma z$.

This can also be written in terms of the inner product of the eigenvectors $\mathbf{X}_+$ (2.4) and $\mathbf{X}^+$ (2.11)

(2.16) $\mathbf{X}^+ \cdot \mathbf{X}_+ = 2 + V$.

The Potential V is not, by itself, an invariant of the solution, so that the inner product is also not an invariant – this also means neither are α, β, γ invariants. However, this all changes for the quadratic exponent $n = 2$, when the potential term is invariant and zero, subject to additional Pythagoras conditions on the dynamical variables – in this case URMT reduces to Pythagoras, with $k = 0$ in (1.0). This is documented in more detail in [5].

So having outlined the basic theory, a complete solution in URMT comprises the following complete set of values:

(2.17) $\{n, x, y, z, k, P, Q, R, \overline{P}, \overline{Q}, \overline{R}, \alpha, \beta, \gamma, K, V\}$, the *complete solution* set.

Nevertheless, obtaining this complete set is a luxury, because it is really only the coordinate equation (1.0) that is of prime concern herein, and this is uniquely specified by the reduced solution set:

(2.18) $\{n, x, y, z, k, \}$, the *reduced solution* set.

It is this reduced set that occupies the bulk of the tabulated solutions in this document, although some complete solutions are given in full, as per (2.17), in Sections (8) and (9).

Lastly, the above focusses on the single, unity eigenvalue, but the unity root matrix **A** (2.3) has two other, non-unity, distinct eigenvalues. Given $\lambda = 1$ then the characteristic equation (2.7) factors, using (2.8), to

(2.19) $(\lambda - 1)(\lambda^2 + \lambda + V) = 0$.

The two eigenvalues are easily obtained by solving the quadratic factor $(\lambda^2 + \lambda + V) = 0$ in the above to give

(2.20) $\lambda = -\frac{1}{2} \pm \frac{\sqrt{(1 - 4V)}}{2}$.

Thus, for a positive, integer, potential energy, $V > 0$, the other two eigenvalues are the complex conjugate pair

(2.21) $\lambda = -\frac{1}{2} \pm i\frac{\sqrt{(4V - 1)}}{2}$, $V > 0$.

By reversing the sign of **A**, these can be made to have a positive real part of ½, and URMT thus now enters the realm of the Riemann Hypothesis – see [6]. One has to admit, this is quite neat to get an equation (1.0) that connects both FLT and the Riemann Hypothesis.

3 A Cubic Exponent Example

This is the classic, cubic example (9,31,70) given in [1]#4 and [2], which has a complete solution (2.16), for non-trivial unity roots (4.2), and currently has the second smallest k-value for a cubic, i.e. $k = 16$.

The coordinate equation (1.0)

(3.0)

$$0 = 9^3 + 31^3 - 70^3 + 16.9.31.70$$

$$n = 3,\ x = 9,\ y = 31,\ z = 70,\ k = 16$$

Dynamical variables P, Q, R and their conjugates $\overline{P}, \overline{Q}, \overline{R}$

(3.1)

$$P = -2,\ Q = -6,\ R = -11$$

$$\overline{P} = +4,\ \overline{Q} = +5,\ \overline{R} = +19.$$

The unity root matrix

(3.1b) $$\mathbf{A} = \begin{pmatrix} 0 & -11 & +5 \\ +19 & 0 & -2 \\ -6 & +4 & 0 \end{pmatrix}.$$

The eigenvector solution

(3.2) $$\mathbf{X}_+ = \begin{pmatrix} 9 \\ 31 \\ 70 \end{pmatrix}.$$

The eigenvector equation ('dynamical equations' in [1]), in matrix form

(3.3) $$\mathbf{AX} = \mathbf{X},\ \begin{pmatrix} 9 \\ 31 \\ 70 \end{pmatrix} = \begin{pmatrix} 0 & -11 & +5 \\ +19 & 0 & -2 \\ -6 & +4 & 0 \end{pmatrix} \begin{pmatrix} 9 \\ 31 \\ 70 \end{pmatrix}.$$

The kinetic term K

(3.4) $$K = P\overline{P} + Q\overline{Q} + R\overline{R} = -247$$

The Potential term V

(3.5) $V = PQR + \overline{P}\overline{Q}\overline{R} = 248$

The Dynamical Conservation Equation

(3.6) $1 = K + V = -247 + 248$.

The divisibility factors α , β and γ

(3.7)

$(1 - P\overline{P}) = \alpha x$, $\alpha = +1$

$(1 - Q\overline{Q}) = \beta y$, $\beta = +1$

$(1 - R\overline{R}) = \gamma z$, $\gamma = +3$

The co-vector $\mathbf{X}^+ = \begin{pmatrix} \alpha & \beta & \gamma \end{pmatrix}$

(3.8) $\mathbf{X}^+ = \begin{pmatrix} 1 & 1 & 3 \end{pmatrix}$

The dual dynamical equations $\mathbf{X}^+\mathbf{A} = \mathbf{X}^+$ in matrix form, unity eigenvalue $C = 1$

(3.9) $\begin{pmatrix} 1 & 1 & 3 \end{pmatrix} \begin{pmatrix} 0 & -11 & +5 \\ +19 & 0 & -2 \\ -6 & +4 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 1 & 3 \end{pmatrix}$.

The Potential Equation in Vector Form

(3.10)

$\mathbf{X}^+ \cdot \mathbf{X}_+ = 2 + V$

$$\begin{pmatrix} 1 & 1 & 3 \end{pmatrix} \begin{pmatrix} 9 \\ 31 \\ 70 \end{pmatrix} = 250 = 2 + 248$$

Unity Root Properties

(3.11)

$P^n \equiv +1 \pmod{x}$, $-2^3 \equiv +1 \pmod{9}$

$Q^n \equiv +1 \pmod{y}$, $-6^3 \equiv +1 \pmod{31}$

$R^n \equiv -1 \pmod{z}$, $-11^3 \equiv -1 \pmod{70}$

$\overline{P}^n \equiv +1 \pmod{x}$, $+4^3 \equiv +1 \pmod{9}$

$\overline{Q}^n \equiv +1 \pmod{y}$, $5^3 \equiv +1 \pmod{31}$

$\overline{R}^n \equiv -1 \pmod{z}$, $+19^3 \equiv -1 \pmod{70}$.

Conjugate Relations

(3.12)
$\overline{P} \equiv P^{n-1} \pmod{x}$, $+4 \equiv (-2)^2 \pmod{9}$
$\overline{Q} \equiv Q^{n-1} \pmod{y}$, $+5 \equiv (-6)^2 \pmod{31}$
$\overline{R} \equiv -R^{n-1} \pmod{z}$, $+19 \equiv -(-11)^2 \pmod{70}$

Since the potential is positive then, by (2.20), the other two eigenvalues are a complex conjugate pair, and using (2.21) these evaluate to

(3.13) $\lambda = -\frac{1}{2} \pm 15.74i$, to 2dps.

4 Unity Roots

It may be of little surprise to the reader, given the subject matter, but unity (or 'primitive') roots are key to this entire subject and, most importantly, obtaining relatively low k-value solutions for (1.0).

There is much in the number theory literature on unity roots under the subject of primitive roots or power-residues – usually to 'prime moduli'. See [7] for example.

The author has also written much on them and their algorithmic determination [1]#4. Also see [3], which gives several conditions any FLT counter-example must satisfy, were it to exist! Of course, whilst Wiles proved no such counter-example exists, the work in [3] is pertinent because the same conditions provide for non-trivial unity roots as opposed to trivial unity roots - both terms are defined next within the context of URMT. Non-trivial unity roots can, essentially, keep the k-values low, see also Section (4.2).

Non-trivial unity roots are tabulated for a range of exponents in Section (11).

4.1 Definition. Trivial root

A unity root P is termed trivial in URMT if, for odd exponent n, modulus x, it is of the form

(4.0) $P^n \equiv 1 \pmod{x}$ and $P \equiv 1 \pmod{x}$.

In other words, if P is of the following form, for arbitrary integer m, including zero:

(4.1) $P = mx + 1$, $m \in \mathbb{Z}$.

This then also applies to Q, i.e.

(4.2) $Q^n \equiv 1 (\text{mod}\, y)$ and $Q \equiv 1\ (\text{mod}\, y)$,

and also the conjugates $\overline{P}$ and $\overline{Q}$.

For unity root *R*, it is considered to be trivial, for odd exponent, modulo *z*, if

(4.3) $R^n \equiv -1 (\text{mod}\, z)$, $R \equiv -1\ (\text{mod}\, z)$,

and, in such a case, the unity root *R* can be written, for some integer *t*, as

(4.4) $R = tx - 1,\ t \in \mathbb{Z}$.

and likewise for the conjugate $\overline{R}$.

4.2 Definition. Non-trivial root

From the definition of a trivial root, then a non-trivial unity root is thus defined as any root that is not trivial, i.e. not congruent to unity, modulo its coordinate *x*, *y* or *z*, but satisfying its unity root definition (2.0) or (2.1).

For example, for cubic exponent $n = 3$, coordinate $x = 7$, the smallest non-trivial unity root is 2, because

(4.5) $2^3 \equiv 1\ (\text{mod}\, 7)$.

The only smaller root is 1, which is trivial by definition, i.e.

(4.6) $1 \equiv 1\ (\text{mod}\, 7)$.

The modulus 7 also has a unity root 4 such that

(4.7) $4^3 \equiv 1\ (\text{mod}\, 7)$.

Together with the single, trivial unity root 1, the set of three unity roots $\{1,2,4\}$ forms a complete set of cubic unity roots, mod 7, and, thereafter, all unity roots are of the following form, for some integer *m*:

(4.8) $\{1 + 7m, 2 + 7m, 4 + 7m\}$.

It is no coincidence that this set comprises three elements, the same as the exponent, $n = 3$, except this is also a consequence of the modulus ($x = 7$) being prime and of a special form discussed very shortly – see also Lagrange's Theorem on Congruences [8]. Furthermore, it is noted that the modulus $x = 7$ is both prime and of the form twice the exponent plus one, i.e., in full, $7 = 2.3 + 1$. If the modulus were prime, but not of this form, it would only have a single, unity (or primitive) root. If the modulus is composite, e.g. 21, then it will also have at least three unity roots (for a cubic exponent) for each factor of the form $2.3+1$. Note, because the exponent ($n = 3$) also divides 21 in this example, this further adds to the unity roots, but this is not actually advantageous – see [3] which discusses this case in more detail.

More formally stated, a coordinate *x* has more than one non-trivial unity root if it is of the following form, for non-zero integers *h* and *m*

(4.9) $x = h(2mn+1)$.

Of course, *h* itself might be composite with yet more factors of the $2mn+1$ form. A good example is $x = 49$, which thus has two equal factors of 7 and, of course, $7 = 2.3+1$ and of the $2mn+1$ form for $m = 1$.

The same remarks apply to *y* and *z*, for some integers *i*,*s* and *j*, *t*, with *s* and *t* not the same as *m* or each other,

(4.10) $y = i(2sn+1)$

(4.11) $z = j(2tn+1)$.

With these facts in hand, the work in [3] concluded that any complete solution (2.16) requires all three coordinates *x*, *y* and *z* to have non-trivial unity roots in order to get a value of *z* close to that of *y* and such that the k-value will consequently be low. However, note that [3] does not talk in terms of k-values or even mention the coordinate equation (1.0). In fact, it talks in terms of a 'root gap', which is the difference between the values of *y* and *z* in (1.0) and how to keep this low. The knock-on effect of a low root gap is that it will keep *k* relatively low. Note, the author admits this is all rather sketchy and needs some rigour – [1]#4 gives more detail, but the reader should also refer to a proper number theory text such as [7] or [8] on the subject of primitive roots and power residues. Lastly, as regards [3], it does refer to a similar equation to (1.0), termed the Modified FLT equation, which is actually a forerunner of the coordinate equation, and also with its own semi-analytic solution heavily reliant upon unity roots.

Thus, combining the above information then, for arbitrary exponent, to obtain a relatively small k-value, the moduli *x*, *y* or *z* must either be prime, or composite, with prime factors of the '$2mn+1$' form as per above.

5 Finding solutions

There are currently two methods for finding solutions to (1.0), the first is just a brute force, trial search, which gives the reduced solution set (2.18), whereas the second method is a more considered, analytic approach, which gives the complete solution (2.16).

5.1 Brute force, trial search

This method simply starts with a specific value for x and n, then steps through values of y, looking for an integer z that satisfies the following congruence:

(5.0) $z^n \equiv x^n + y^n \bmod(xyz)$.

with z obtained, k is then obtained from (1.2).

Using this reduced solution $\{n, x, y, z, k, \}$ (2.18), a 'complete solution' (2.17) can then be obtained using the algorithm given in Section (5.2.2).

This brute force method thus gives solutions (2.18) to the coordinate equation (1.0), but it does not give the unity roots, i.e. not the complete solution (2.17). Indeed, further work is required to obtain the unity roots and, currently, this has not yet been documented – it is in progress however.

Of course, whilst the brute force method is applicable to any arbitrary x and y, sensible values for x and y can be chosen such that they give non-trivial unity roots, i.e. one or both are of the $2mn+1$ form (4.2).

Whilst the brute force method is essentially 'quick and dirty', it does act as a good, independent check on the more esoteric, semi-analytic solution given next.

5.2 Semi-analytic solution

This method would be wholly analytic, i.e. the solutions given by a deterministic equation, as per Pythagoras [5], were it not for the fact that non-trivial unity roots, Section (4.2), can only be obtained algorithmically [1]#4. It is hardly surprising there is no analytic solution since this could then be used to determine k analytically from the solution for x, y, z using (1.2). This would then mean that one might relatively easily be able to determine k is never zero, hence a simple and direct proof of FLT – no such chance!

This method does have the advantage over the brute force method in that it can produce some relatively low k-values for large exponents, albeit, for small x, the brute force method is quicker.

5.2.1 Algorithmic Determination of Solutions

The steps in determining a complete solution, for an arbitrary x and y, if it exists, are given shortly below. However, it is important to note beforehand that many low k-value solutions can be found that do not follow the numeric constraints given here, for example, neither x nor y need necessarily be of the $x = h(2mn+1)$ and $y = i(2sn+1)$ forms, (4.9) and (4.10) respectively. A particularly notable example is the 'x plus y' solution, Section (5.3), i.e. $z = x + y$ (5.28), which has a constant k-value of three for cubic exponent, and all values of x and y. Furthermore, $k = 3$ is actually the record lowest k-value yet found for any exponent. Nevertheless, when considering studying (1) in relation to FLT, the value of z as the sum of x and y is still too large for any k-value less than three, for higher order exponents.

5.2.1.1 Choose an exponent n

In accordance with FLT, barring the singular, even, quartic exponent $n = 4$, only odd, prime exponents are generally of interest, i.e. odd, prime n, $n \geq 3$ (1.1b).

5.2.1.2 Choose x and y

Firstly, whilst x and y can be completely arbitrary, non-zero, it is only really worth selecting those positive, non-zero integers x,y that satisfy both (1.1) and possess non-trivial unity roots in accordance with Section (4.2).

Start with a lowest value of x not less than two, and y greater than x, i.e. $2 \leq x < y$ (1.1b). In fact, for non-trivial unity roots, the cubic exponent, $n = 3$, with $x = 7$, is a good starting point since $x = h(2mn+1)$ (4.9), where the arbitrary factor h is 1, so too m - see Section (4.2).

Selection of y is then such that it too is co-prime to x (1.1b) and, preferably also of the form $y = i(2sn+1)$ (4.10). For a cubic exponent, with $x = 7$, the next value is thus $y = 13$ where $i = 1$ and $s = 2$. This case is the classic cubic example thoroughly detailed in [1]#4, and has the solution (7,13,635) with $k = 4431$, see Section (7.3.6).

5.2.1.3 Calculate the unity roots for x and y

Unity roots are defined by (2.0) and (2.1). For an arbitrary exponent, and coordinate (x or y here), there is no simple analytic solution to determine them and they have to be found algorithmically. The algorithm is fully documented in [3] which is freely available for download, so the algorithm will not be detailed herein. Suffice to say, the quickest method is generally brute force. For example, for coordinate x, try each value $\overline{P}$ (Note *) between $1 \leq \overline{P} < x-1$ that satisfies $\overline{P}^n \equiv 1 \pmod{x}$ (2.0). Evidently the trivial root '1' is always a root, but there are others in the range $1 < \overline{P} < x-1$ when x has the general form $x = h(2mn+1)$ (4.9). Similar remarks apply to the unity roots of y.

Note *. The choice of $\overline{P}$ instead of P here (both of which are unity roots of x) is related to the next step of the algorithm when determining z.

Whilst every x has a trivial unity root $\overline{P}$, where $\overline{P} \equiv 1 \pmod{x}$ (4.0), it is only non-trivial unity roots that are of real interest – see the above on choosing x and y to obtain non-trivial roots.

For example, the full set of three unity roots $\overline{P}$ for the cubic exponent, $x = 7$

(5.1) $x = 7$, $n = 3$, $\overline{P} \in \{1,2,4\}$, $1 \le \overline{P} < x - 1$

and for $y = 13$, unity root *Q*,

(5.2) $y = 13$, $n = 3$, $Q \in \{1,3,9\}$, $1 \le Q < y - 1$.

The non-trivial unity roots of *x* and *y* are thus $\{2,4\}$ and $\{3,9\}$ respectively.

This is not quite the end of unity root determination because it is preferable to adjust $\overline{P}$ to be negative in the hope of keeping the value for *z*, calculated next, relatively small, i.e. close to *x* and *y* within the constraint $x < y < z$ (1.1).

(5.3) $\overline{P} \to \overline{P} - x$ such that $-(x-1) < \overline{P} < -1$.

Thus, the example set of unity roots for $\overline{P}$, given above, is now adjusted to the all negative set

(5.4) $x = 7$, $n = 3$, $\overline{P} \in \{-6,-5,-3\}$, $-(x-1) < \overline{P} < -1$.

Note that this may well make *z* initially negative, and the above negation of $\overline{P}$ can then be undone when adjusting *z* to positive - this detail is given in the next step. It is also possible to adjust *Q* negative instead of $\overline{P}$ and, in fact, might be wiser given how *z* is formed (2.5d). Nevertheless, the author's current algorithm adjusts $\overline{P}$ to negative as it was early convention to have the conjugate dynamical variables $\overline{P}, \overline{Q}, \overline{R}$ negative, or at least the opposite sign to the standard forms P, Q, R where possible. Admittedly though, this is purely convention with no mathematical basis!

5.2.1.4 Determination of *z*

Having chosen *n*, *x*, *y* and formed a set of unity roots, then the next step is to calculate *z* from the 'dynamical equation'

$z = Qx + \overline{P}y$ (2.5d).

A separate *z* is calculated for each of the non-trivial unity roots. Thus, in the above example with four non-trivial roots, two each for *x* and *y*, there are four possible *z* values. The reader may well like to include the trivial root '1' for either/or both *x* and *y* for experimentation – for a cubic exponent, this should always give the 'x plus y' solution (5.3).

Given $\overline{P}$ was adjusted negative in the previous example, it is possible that *z* is negative or at least $z \le y$, as calculated above. In this case, it needs adjusting so that $y < z$ in accordance with (1.1a). The pseudo code to do this adjustment is as follows:

(5.5)

Repeat until $y < z$
if $z \le y$ then {
$z \to z + xy$
if $\overline{P} < Q$ then $\overline{P} \to \overline{P} + x$
else $Q \to Q + y$
}
end

This adjustment may well push $\overline{P}$, originally adjusted negative as in the previous step, back to positive. It is harmless, but the two combined steps may seem pointless.

5.2.1.5 Perform the z congruence test.

So, at this stage, an exponent *n*, coordinates *x*,*y* and *z*, and two unity roots $\overline{P}$ and Q are all defined and satisfy their desired criteria. The next step is thus to determine if the three coordinates satisfy the 'coordinate equation' (1.0). To do this the following congruence is used, derived from (1.0):

(5.6) $z^n - (x^n + y^n) \equiv 0 \pmod{xyz}$.

Note that this is similar to computing the k-value using (1.2), and seeing if it evaluates to an integer, i.e. does xyz divide $z^n - (x^n + y^n)$ without remainder?

This test can also be divided into the following three, individual tests:

(5.7)

$z^n \equiv x^n \pmod{y}$

$z^n \equiv y^n \pmod{x}$

$x^n \equiv -y^n \pmod{z}$.

These congruences are simple to compute without the exponential terms growing too large because, for example, the first can be computed equivalently as

(5.8) $z^n \equiv x^n \pmod{y} = [z \pmod{y}]^n \equiv x^n \pmod{y}$.

Similarly, if the exponent is odd of the form $n = 2p + 1$, $p \ge 1$ then

(5.9) $[x^2 \bmod(z)]^p x \equiv -[y^2 \bmod(z)]^p y \pmod{z}$

and so $x^2 \bmod(z)$ and $y^2 \bmod(z)$ can be pre-computed as number less than *z*, and then raised to a power *p*, less than half that of *n*, etc.

So, however the test is actually computed, if $z^n - (x^n + y^n) \equiv 0 \pmod{xyz}$ then x, y, z is a solution, and is not a solution if there is any non-zero remainder $\pmod{xyz}$.

If the congruence test passes, proceed to obtaining the complete solution, as given in Section (5.2.2).

And if the test fails...

There is absolutely no guarantee the above congruence is satisfied for the first value of *z* calculated. Indeed, given *x* and *y* were relatively arbitrarily chosen, the test will usually fail for any pair ($\overline{P}$,*Q*) of the unity roots selected – unless the trivial unity roots, $\overline{P} = 1$ and $Q = 1$, are used, then invariably the $z = x + y$ solution appears (5.3).

However, in a continued attempt to find a solution, the value of *z* can now be varied, i.e. increased upward whilst keeping *x* and *y* constant, albeit this will mean any solution will generally have a large k-value, i.e. k increases as *z* increases (for large *z*) – a bit of analysis follows to show this more rigorously:

For large *z*, i.e.

(5.10) $z >> y > x$

then, by (1.0),

(5.11) $z^n \approx kxyz$,

and thus re-arranging gives

(5.12) $k \approx \dfrac{z^{n-1}}{xy}$, $z >> y > x$.

Since $y > x$ then $y^2 > xy$ and inverting means $\dfrac{1}{y^2} < \dfrac{1}{xy}$ thus

(5.13) $k \approx \dfrac{z^{n-1}}{xy} > \dfrac{z^{n-1}}{y^2}$,

and so from the above, large *z* approximation for *k*

(5.14) $k > \dfrac{z^{n-1}}{y^2}$ for $z >> y > x$.

Defining *z* in terms of *y* and a factor *f* as follows:

(5.15) $z = fy$, $f >> 1$,

then it follows that

(5.16) $k > fy^{n-3}$, $f >> 1$.

Thus, for large *z* (or large *f*) satisfying $z >> y > x$ and $z = fy$, then *k* is at least greater than *f*. In particular, for a cubic exponent,

(5.17) $k > f$, $n = 3$ $f >> 1$.

Thus, all in all, increasing *z* generally results in a larger k-value.

Returning to the algorithmic determination of *z*, and thus a solution to (1.0), if the first value of *z* fails, it can be increased upward in steps of the multiple xy, exactly as before when adjusting *z* from negative to $y < z$. One of the unity roots $\overline{P}$ and Q is also adjusted simultaneously to maintain the dynamical equation $z = Qx + \overline{P}y$ (2.5d).

(5.18)

If the congruence test fails then repeat until
the test passes or *z* exceeds some predefined limit
{
$z \to z + xy$
if $\overline{P} < Q$ then $\overline{P} \to \overline{P} + x$
else $Q \to Q + y$
}

This process is repeated until the test either passes or *z* reaches a specific limit (more below). The cubic solution (7,13,635) is a good example where the loop needs to iterate six times, since at each loop *z* increases by 91 ($xy = 7 \times 13$) starting at 89.

It is evident from the k-value analysis above that, when *z* is much greater than *y*, the k-value is no longer small enough to warrant much interest. It is worth, therefore, perhaps basing a limit for *z* in terms of the maximum acceptable k-value.

From the earlier approximation for *k*, repeated below,

(5.19) $k \approx \dfrac{z^{n-1}}{xy}$, $z >> y > x$.

then rearranging gives *z* in terms of *k* and the fixed product xy, i.e.

(5.20) $z \approx (kxy)^{1/(n-1)}$, $z >> y > x$.

For the cubic exponent therefore

(5.21) $z \approx \sqrt{kxy}$, $n = 3$, $z >> y > x$.

Thus, imposing a limit on k can impose a limit on z under the conditions $z >> y > x$.

The aforementioned (7,13,635) solution gives an exact k-value $k = 4431$, and substituting for k, x and y gives the z limit as

(5.22) $z = 634.99$ to 2dps, $n = 3$, $x = 7$, $y = 13$, $k = 4431$,

Since z is actually 635, this is evidently a very good approximation for z – albeit k was pre-determined. This k-value is admittedly large, but not so large given the huge k-values tabulated in Section (7). In brief, it is largely up to the reader to set the limit according to the search range of interest but some guidance is given by the above analysis. In fact, the tabulated values in Section (7) were actually limited by the rather more restrictive criterion (7.1), and did not use the above analysis!

Lastly, just to reiterate, whilst for odd exponent there is always the 'x plus y' solution, Section (5.3), the author has no proof or method of determination whether a solution to the coordinate equation (1.0) can always be found for arbitrary x and y.

5.2.2 Obtaining the Complete Solution

The complete solution (2.17) can be obtained from the reduced solution (2.18), by the calculations given in this section. The latter, reduced solution, is obtained either by brute force calculation, Section (5.1), or by the semi-analytic method, using the algorithm detailed in Section (5.2.1).

This algorithm only requires the x, y, z solution, and not the Q and $\overline{P}$ unity roots, or any others, even though they are determined in the semi-analytic algorithm given above in Section (5.2.1). However, the brute force method, Section (5.1), only produces x, y, z, and not Q and $\overline{P}$, so the algorithm is suitable for both brute force, and semi-analytic methods.

The solution assumes x, y, z satisfy the gcd, co-primality in pairs condition (1.1b) but, in general, the solution does not have to be restricted to the stringent conditions on x and y given in the previous algorithm, i.e. if one has a solution x, y, z to (1.0), then the calculations given next should suffice to find the unity roots P, Q, R, $\overline{P}, \overline{Q}, \overline{R}$ and dual variables α, β, γ. In addition, the Kinetic and Potential terms, K and V, can then simply be derived from the unity roots using (2.6).

The algorithmic steps to obtain a complete solution, starting with the reduced solution for x, y, z, are given below.

5.2.2.1 Obtain unity roots Q and $\overline{P}$

This step can be skipped if Q and $\overline{P}$ have already been obtained using the semi-analytic algorithm, Section (5.2.1).

These two unity roots are obtained by solving the following linear Diophantine equation (LDE), which is just the dynamical equation for z:

$z = Qx + \overline{P}y$ (2.5d).

Given that x, y, z is a solution to (1.0), this will always have a solution for Q and $\overline{P}$. Note that this is a non-unique solution for some integer s, with a general solution given by

(5.23)

$$\overline{P} = \overline{P}_{s=0} - sx$$

$$Q = Q_{s=0} + sy\,.$$

Indeed, given this parametric freedom, the values for Q and $\overline{P}$ can be adjusted by varying s to make the computations more manageable, e.g. possibly stop them from growing too large and/or avoid numerical overflow. Note however, if one root becomes more positive, the other becomes more negative.

5.2.2.2 Obtain unity roots P and $\overline{Q}$

With a value for Q and $\overline{P}$ fixed, the conjugate relations (2.2) are then used to obtain P and $\overline{Q}$, i.e.

(5.24)

$$P \equiv \overline{P}^{n-1} \pmod{x}$$

$$\overline{Q} \equiv Q^{n-1} \pmod{y}\,.$$

5.2.2.3 Obtain unity roots R and $\overline{R}$

With $\overline{Q}$ and P obtained, the remaining two unity roots, R and $\overline{R}$ are calculated by rearranging the dynamical equations (2.5b) and (2.5c) to give

(5.26)

$$R = \frac{(x - \overline{Q}z)}{y} \quad (2.5b)$$

$\overline{R} = \dfrac{(y - Pz)}{x}$ (2.5c).

5.2.2.4 Calculate the dual variables

At this stage x, y, z and all six unity roots $P, Q, R, \overline{P}, \overline{Q}, \overline{R}$ gave been obtained. The dual variables α, β, γ are then simply obtained from them by rearranging equations (2.10) to give

(5.27)

$$\alpha = \frac{(1 - P\overline{P})}{x}$$

$$\beta = \frac{(1 - Q\overline{Q})}{y}$$

$$\gamma = \frac{(1 - R\overline{R})}{z}.$$

This completes the set of unknowns, all bar the Kinetic and Potential terms, K and V respectively, which can easily be obtained from (2.6). The Potential equation (2.15) can also be verified using this complete solution.

5.3 The 'x plus y' solution.

There is a very simple, analytic solution, known as the 'x plus y' solution, defined for arbitrary, non-zero coordinates x and y, by

(5.28) $z = x + y$,

and valid for all odd exponents $n = 2p + 1$, $p \in \mathbb{Z}$, $p \geq 1$. This solution is not documented herein primarily because it is a completely solved problem with a full analytic solution, and is detailed further in [1]#1. Interestingly, for the cubic, it is simply shown that the k-value is independent of x and y, and is always 3. This is a very low k-value (see further below) compared with all other solutions (in fact it is the lowest), but its unity roots are trivial, Section (4.1), and the complete solution (2.17) is therefore of little interest other than a curio. In fact, not only are they trivial, they are all of unity magnitude, i.e. +/-1. For higher exponents, $n \geq 5$, k grows larger than 3, and is a more involved polynomial function in x and y of order $n - 3$, hence it is constant for $n = 3$.

5.3.1 Extended forms

Note that there are other forms of similar solutions to (5.28), for trivial roots Q and $\overline{P}$, known as 'repeat' or 'extended' solutions. These are constructed using the z dynamical equation (2.5d), reproduced below,

$z = Qx + \overline{P}y$ (2.5d).

If Q and $\overline{P}$ are trivial then, by definition, e.g. (4.1), they are of the following form for some integers *s* and *t*:

(5.29) $\overline{P} = sx + 1$, $Q = ty + 1$,

and so the equation for *z* (above) can be written as

(5.30) $z = x + y + (s + t)xy$.

This is now similar to (5.28), but with an additional term $(s + t)xy$. The consequence of this is that not only is $z = x + y$ a valid solution but, for some trivial roots, so can (5.30) also be a solution. However, it is most definitely not the case for every *s,t* combination in (5.30), far from it. Nevertheless, there are repeat (or extended) solutions of $z = x + y$. A notable example is the n=13, (3,5,53) solution given in table (7.7.2), which is actually of the form $z = (3 + 5) + 3.(3.5)$, where $x = 3$, $y = 5$, $xy = 15$, $s + t = 3$. Hence, the smaller (k value) solution is $z = (3 + 5)$, which is not actually tabulated herein because it is directly of the form $z = x + y$, with trivial unity roots $\overline{P} = 1$, $Q = 1$ for zero *s,t*. Note, this $z = 8$ example has a k-value of 4,571,112,637, i.e. a mere 10 digits, as opposed to the 20 digit value for the (3,5,53) solution given in table (7.7.2). In both these cases, i.e. (3,5,8) and (3,5,53), this k-value is still too high to be of any real note, and this is generally characteristic of k-values for trivial unity roots, excepting the cubic 'x plus y' solution, which, as mentioned, has a constant $k = 3$ value, but none lower. Of course, this k-value is actually still the lowest seen. However, it is for the single, cubic exponent and, ultimately, it would be nice to find a k-value of one or two – whether this is theoretically possible remains unknown. If the reader could prove k is non-zero for all exponents, a simple and direct proof of FLT would be theirs!

Lastly, the example solution discussed above, i.e. n=13, (3,5,53), is not the only solution of the extended form (5.30), and there are others tabulated, which have not been highlighted.

6 What is a 'low' k-value?

The phrase 'low k-value' has been bandied around without any real justification. Indeed, it is slightly ethereal but comes from the empirical evidence.

Before continuing, note that the 'x plus y' solution, documented above, is generally discarded in the tabulated solutions (with some exceptions), because it has trivial unity roots. However, for the single, cubic exponent, it does have an exceptionally small and constant k-value of three. Nevertheless, because this is a one-off exponent, with trivial unity roots, it is disregarded.

From a scan of the early results in Section (7), it would seem that a relatively low k-value is anything less than one thousand, and this is really for the smallest, cubic exponent, $n = 3$. Looking at exponents, $n \geq 5$, shows that even a value $k < 1000$ seems non-existent, albeit that is based upon the small set of results tabulated. Nevertheless, for $n = 5$, the quintic solution (5,11,31) gives a ‘low’ value k=16695, which is low in comparison with the other values tabulated. Indeed, for $n \geq 5$, the k-value is generally six digits or larger. For exponents $n \geq 7$, the k-value is well into 10 digits. So, whilst very subjective, one thing seems certain, to find a value under $k \leq 10$, for solutions that adhere to conditions (1.1), would be quite some find. One would also hopefully appreciate that a value close to unity is almost impossible. Of course, a value of zero is impossible in accordance with the proof of FLT.

If the conditions (1.1) are relaxed so that z is arbitrary, but still a non-zero, positive integer (it may well be unity), then it is possible to get smaller k-values.

7 Tables

The tables in this section list the reduced solutions to the coordinate equation, as determined using the brute force method, Section (5.1), with some restrictions on the search range limits, see further.

7.1 General

The tables are listed in exponent order, cubic exponent first, for prime exponents only.

Outside of FLT, i.e. non-zero *k*, even exponents have their own, distinct solutions and, indeed, the $n = 4$ case still requires a proof of no solutions when considering FLT, since the 'prime' exponent $n = 2$ does have solutions. As such, some even exponent, $n = 4$, solutions are given in Section (1.2).

Large k-values, i.e. those above 8 digits, are generally written in groups of four digits, as per a credit card number, rather than the more usual grouping in threes. This is mainly because the computer program that generates them sub-divides large numbers into elements 0-9999. In this way, such a four-digit number can be squared whilst not overflowing the 32-bit signed range, maximum value 2^31-1. This also means such eight digit, squared numbers can be added multiple times without overflow, albeit not indefinitely, of course.

Not all large k-values are listed in full, instead only the number of digits is specified. There is no specific reason for this other than, originally, it was not the intention to list any k-values more than eight or so digits, but some concession was made to supply the k-values for independent checking.

All computations have been done using the author's proprietary large number library (C language). Whilst it has a good pedigree, i.e. it has been used and developed since circa 2000 in the search for 1000+ digit primes, it can never be guaranteed to be bug-free. It is the intention to release this library with full source-code sometime, and the only really barrier is tidying, documentation and automated testing – plus there are other libraries out there. If demand is sufficient (email request), the author will endeavour to release it.

7.2 Search Range Limitations

The solutions exclude $x = 1$, and start at $x = 2$. However, outside of FLT, but within the scope of the coordinate equation, the value of coordinate x may well also be unity. Likewise, the exponent starts at $n = 3$ and excludes the quadratic exponent, $n = 2$, especially since, for $k = 0$, the coordinate equation (1.0) is just Pythagoras, and therefore widely documented, not least in URMT, [5].

The solutions rather arbitrarily limit the z value to a factor ten of the sum of x and y, i.e.

(7.1) $z \le 10(x + y)$.

For small x and y, this means the search is slightly restricted, but does keep the k-value low. In truth, and not mentioned herein, see [3] (Note *), for really low k-values, z should be less than the sum of x and y, i.e. an ideal range of solutions would be $x < y < z < (x + y)$. See also Section (5.2.1.5) for some analysis on k given a large z.

Note *. The reference [3] actually talks in terms of a 'Root Gap', abbreviated to 'Rg', where this is the difference between z and y, i.e. $z - y$ - the coordinates x, y, z are also replaced by a, b, c in [3].

The reader might like to search all $x = 1$ solutions, and/or also allow arbitrary $z > 0$ for a complete study of k-values. There are a lot more solutions for $x = 1$, and arbitrary, non-zero z, with accordingly lower k-values, but these solutions are not in the spirit of FLT or URMT!

The upper limit to which y has been searched, for a specific value of x, varies slightly, from 2000 minimum up to 10000. Again, this is rather arbitrary and primarily decided upon computational time taken to do the search. At the moment, all searches are relatively fast with around 15 minutes maximum required for an exponent such as $n = 13$, for prime x – prime x slows the computation because there are no gcd candidates (1.1) that can be quickly eliminated, hence the brute force method search slogs through all y in the search range.

The (x) denotes that the solution is an 'extended form' of the 'x plus y' solution, see Section (5.3.1).

7.3 n=3

7.3.1 n=3, x=2

n	x	y	z	k
3	2	3	35 (x)	204
3	2	7	39	108
3	2	13	45	76
3	2	19	109	311
3	2	171	1211	4276
3	2	229	1333	3860
3	2	385	2587	8663
3	2	633	1505	1656
3	2	637	3711	10755
3	2	3723	18923	47724
3	2	6191	21527	36536

3	2	7771	45291	131316
3	2	to 9999	none	

(x) denotes that the solution is an 'extended form' of the 'x plus y' solution, see Section (5.3.1).

7.3.2 n=3, x=3

n divides x

n	x	y	z	k
3	3	5	38 (x)	96
3	3	13	139	495
3	3	112	643	1224
3	3	122	995	2700
3	3	247	763	759
3	3	673	1924	1755
3	3	892	7699	22116
3	3	4699	43183	132111
3	3	6743	19418	17859
3	3	893 to 9999		

7.3.3 n=3, x=4

n	x	y	z	k
3	4	9	61	103
3	4	175	379	185
3	4	663	1687	1008
3	4	2169	15601	27978
3	4	2679	16783	26178
3	4	6251	14715	7996
3	4	8105	46269	65679
3	4	8106 to 9999	none	

7.3.4 n=3, x=5

n	x	y	z	k
3	5	7	52	77
3	5	7	117 (x)	391
3	5	67	252	186
3	5	99	434	376
3	5	2083	10248	9999
3	5	2128	9973	9257
3	5	2416	7861	4967
3	5	2709	8909	5695
3	5	3361	7821	3351
3	5	3819	10664	5682

3	5	4251	10016	4359
3	5	5692	51597	93418
3	5	4252 to 9999	none	

(x) denotes that the solution is an ‘extended form’ of the ‘x plus y’ solution, see Section (5.3.1).

7.3.5 n=3, x=6

n divides x

n	x	y	z	k
3	6	91	409	303
3	6	92 to 9999	none	

7.3.6 n=3, x=7 special solution

Prime x, $2mn+1$ form, 7 = 2.3 + 1, n=3, m=2

n	x	y	z	k
3	7	8	95	161
3	7	9	67	71
3	7	13	635	4431 Note *
3	7	18	247	484
3	7	78	151	36
3	7	122	817	779
3	7	148	403	149
3	7	153	976	886
3	7	247	1502	1299
3	7	248	2159	2681
3	7	1287	6079	4063
3	7	1668	2479	366
3	7	1833	6352	3069
3	7	1961	17497	22271
3	7	2074	5209	1751
3	7	2078	16565	18827
3	7	2409	22348	29580
3	7	3782	4887	484
3	7	4295	12987	5407
3	7	5356	45403	54893
3	7	5738	15777	5899
3	7	6633	12118	2644
3	7	7081	14072	3486
3	7	7265	44082	38040
3	7	8465	82602	115024
3	7	9417	20596	5820
3	7	9579	49111	35703

3	7	9621	26116	9621
3	7	9622 to 9999	none	

Note *. This (7,13,635) solution is detailed in full in [1]#4 as an example. However, it is actually outside the scope of the tabulated solutions herein, where *z* has been rather arbitrarily limited to $z \le 10(x+y)$ (7.1).

7.3.7 n=3, x=8

n	x	y	z	k
3	8	97	585	439
3	8	1141	5517	3305
3	8	1333	5061	2358
3	8	4009	9081	2350
3	8	7007	50135	44717
3	8	7169	25153	10776
3	8	1334 to 9999	none	

7.3.8 n=3, x=9 special solution

n divides x twice, i.e. 9=3.3

n	x	y	z	k
3	9	13	133	151
3	9	31	70	16 Note *
3	9	38	329	316
3	9	103	208	41
3	9	637	1054	151
3	9	946	9139	9799
3	9	1271	5216	2344
3	9	1279	1978	248
3	9	1390	11359	10295
3	9	1463	4376	1400
3	9	3943	5548	556
3	9	5491	53185	57175
3	9	6140	34529	21454
3	9	6289	8554	779
3	9	6290 to 9999	none	

Note *. This cubic (9,31,70) solution is a special solution with a low k-value, and the complete solution is tabulated again in Section (8.1.1).

7.3.9 n=3, x=10

n=3, x=10, y=351, z=2071, k=1216

7.3.10 n=3, x=11

n	x	y	z	k
3	11	525	2186	816
3	11	1090	6381	3379
3	11	3094	13203	5056
3	11	4709	33892	22116
3	11	4710 to 9999	none	

7.3.11 n=3, x=12

n	x	y	z	k
3	12	4429	26689	13341
3	12	4430 to 5000	none	

7.3.12 n=3, x=13 special solution

13 = 4.3 + 1, n=3, l=2

n	x	y	z	k
3	13	14	61	20 Note *
3	13	35	313	215
3	13	203	999	375
3	13	271	436	41
3	13	739	1204	116
3	13	855	1663	215
3	13	1962	3871	511
3	13	3087	9700	2269
3	13	3182	16891	6851
3	13	3377	23553	12599
3	13	3404	32949	24506
3	13	8135	27468	6949
3	13	8136 to 9999	none	

Note *. This cubic (13,14,61) solution is a special solution with a low k-value, and the complete solution is tabulated again in Section (8.1.1).

7.3.13 n=3, x=14

Composite x has the $2mn+1$ factor 7, i.e. x=2(2.3+1), n=3, m=1

n	x	y	z	k
3	14	19	97	35 Note *

3	14	367	2127	876
3	14	577	3351	1383
3	14	5211	17195	3940
3	14	5212 to 9999	none	

Note *. This cubic (13,14,61) solution is a special solution with a low k-value, and the complete solution is tabulated again in Section (8.1.1).

7.3.14 n=3, x=15

There are no solutions (15,y,z) from y=16 to 9999 for x=15, n=3.

7.3.15 n=3, x=16

n	x	y	z	k
3	16	1579	11803	5501
3	16	2331	9451	2359
3	16	2332 to 9999	none	

7.3.16 n=3, x=17

Only one solution to y=5000

n	x	y	z	k
3	17	633	3965	1455
3	17	5768	15305	2261
3	17	5769 to 9999	none	

7.3.17 n=3, x=18

n divides x twice

n=3, x=18, y=19, z=259, k=196

7.3.18 n=3, x=19 low k-value 6

x = $2mn+1$ form, x=6.3+1, n=3, m=3

n	x	y	z	k
3	19	21	52	6 note *
3	19	325	2644	1130
3	19	554	819	44
3	19	1355	5139	1007
3	19	1505	13779	6631
3	19	2021	12380	3974
3	19	3365	19364	5834
3	19	3789	18904	4924
3	19	7498	30931	6620

3	19	7499 to 9999	none	

Note *. This cubic (19,21,52) solution is a special solution with a low k-value, and the complete solution is tabulated again in both Section (8.1.1) and also (10.1) since it is, in fact, the smallest known k-value for non-trivial, cubic unity roots.

7.3.19 n=3, x=20

n=3, x=20, y=2923, z=16023, k=4365

7.3.20 n=3, x=21

n divides x

Composite x has the $2mn+1$ factor 7, i.e. x=3(2.3+1), n=3, m=1.

n=3, x=21, y=9703, z=62308, k=18981

7.3.21 n=3, x=22

There are no solutions (22,y,z) from y=23 to 9999 for x=23, n=3.

7.3.22 n=3, x=23

n=3, x=23, y=133, z=1467, k=703

7.3.23 n=3, x=24

There are no solutions (24,y,z) from y=25 to 9999 for x=24, n=3.

7.3.24 n=3, x=25

n=3, x=25, y=5983, z=30133, k=6023

7.3.25 n=3, x=26

There are no solutions (26,y,z) from y=27 to 9999 for x=26, n=3.

7.3.26 n=3, x=27

The x-value is divisible by the exponent three times.

n	x	y	z	k
3	27	1729 Note *	11332	2741
3	27	1730 to 9999	none	

Note *. The composite y=1729 has three $2mn+1$ factors 7,13,19 i.e. x= (2.3+1) (4.3+1) (6.3+1).

This *y* value has 26 non-trivial unity roots, see Section (11.1).

7.3.27 n=3, x=28

Composite *x* has the $2mn+1$ factor 7, i.e. x=4(2.3+1), n=3, m=1.

n	x	y	z	k
3	28	209	633	66
3	28	210 to 9999	none	

7.3.28 n=3, x=29

There are no solutions (29,y,z) from y=30 to 9999 for x=29, n=3.

7.3.29 n=3, x=30

There are no solutions (30,y,z) from y=31 to 9999 for x=30, n=3.

7.3.30 n=3, x=31

Prime x, $2mn+1$ form, i.e. x=(2.5.3+1), n=3, m=5.

n	x	y	z	k
3	31	2249	22002	6963
3	31	2250 to 9999	none	

7.3.31 n=3, x=32

There are no solutions (32,y,z) from y=33 to 9999 for x=32, n=3.

7.3.32 n=3, x=33

n divides x

n=3, x=33, y=511, z=5032, k=1500

7.3.33 n=3, x=34

There are no solutions (34,y,z) from y=35 to 9999 for x=34, n=3.

7.3.34 n=3, x=35

Composite *x* has the $2mn+1$ factor 7, i.e. x=5(2.3+1), n=3, m=1.

n	x	y	z	k
3	35	4516	6771	204
3	35	5161	46881	12151
3	35	5162 to 9999	none	

7.3.35 n=3, x=36

There are no solutions (36,y,z) from y=37 to 9999 for x=36, n=3.

7.3.36 n=3, x=37

Prime x

n=3, x=37, y=767, z=7353, k=1903

7.3.37 n=3, x=38

Composite *x* has the $2mn+1$ factor 19, i.e. x=2(2.3.3+1), n=3, m=3.

n	x	y	z	k
3	38	367	845	47
3	38	1417	3345	192
3	38	1418 to 9999	none	

7.3.38 n=3, x=39

n divides x

Composite *x* has the $2mn+1$ factor 13, i.e. x=3(2.2.3+1), n=3, m=2.

n	x	y	z	k
3	39	76	619	129
3	39	905	2234	132
3	39	1033	8884	1956
3	39	1034 to 9999	none	

7.3.39 n=3, x=40

There are no solutions (40,y,z) from y=41 to 9999 for x=40, n=3.

7.3.40 n=3, x=41

Prime x

There are no solutions (41,y,z) from y=42 to 9999 for x=41, n=3.

7.3.41 n=3, x=42

n divides x

There are no solutions (42,y,z) from y=43 to 9999 for x=42, n=3.

7.3.42 n=3, x=43

Prime x, $2mn+1$ form, i.e. x=(2.7.3+1), n=3, m=5.

n	x	y	z	k
3	43	1764	3691	160
3	43	1765 to 9999	none	

7.3.43 n=3, x=44

There are no solutions (44,y,z) from y=45 to 9999 for x=44, n=3.

7.3.44 n=3, x=45

n divides x twice

Composite x has the $2mn+1$ factor 13, i.e. x=3(2.2.3+1), n=3, m=2.

n	x	y	z	k
3	45	151	931	127
3	45	203	1118	136
3	45	204 to 9999	none	

7.3.45 n=3, x=46

There are no solutions (46,y,z) from y=47 to 9999 for x=46, n=3.

7.3.46 n=3, x=47

Prime x

There are no solutions (47,y,z) from y=48 to 9999 for x=47, n=3.

7.3.47 n=3, x=48

n divides x

Composite x has the $2mn+1$ factor 13, i.e. x=3(2.2.3+1), n=3, m=2.

n=3, x=48, y=4489, z=38041, k=6705

7.3.48 n=3, x=49

Composite x has the $2mn+1$ factor 7 twice, i.e. x=(2.1.3+1) (2.1.3+1), n=3, m=1.

n=3, x=49, y=1322, z=9243, k=1315

7.3.49 n=3, x=52

Composite x has the $2mn+1$ factor 13, i.e. x=4(2.2.3+1), n=3, m=2.

n=3, x=52, y=4559, z=34503, k=5010

7.3.50 n=3, x=53 to 62

There are no solutions for x in the range 53 to 62, and y in the range x+1 to 5000.

7.3.51 n=3, x=63

n divides x twice

Composite x has the $2mn+1$ factor 7, i.e. x=3.3(2.3+1), n=3, m=1.

This x value has eight, non-trivial unity roots, see Section (11.1).

n	x	y	z	k
3	63	73	1258	344
3	63	737	1460	40
3	63	738 to 9999	none	

7.3.52 n=3, x=64

There are no solutions for x=64, y in the range 65 to 9999.

7.3.53 n=3, x=65

n=3, x=65, y=8127, z=12602, k=220

7.3.54 n=3, x=66

There are no solutions for x=66, y in the range 67 to 9999.

7.3.55 n=3, x=67

n=3, x=67, y=234, z=2293, k=335
n=3, x=67, y=3878, z=21345, k=1743

7.3.56 n=3, x=68 to 90

n=3, x=76, y=8901, z=80737, k=9623
n=3, x=77, y=9503, z=17060, k=329
n=3, x=81, y=5894, z=18881, k=724

7.3.57 n=3, x=91 low k-value 15 special solution

Composite x has two $2mn+1$ factors 7 and 13, i.e. x=(2.2.3+1)(2.1.3+1), n=3, m=1, 2.

This x value has eight, non-trivial unity roots, see Section (11.1).

n=3, x=91, y=185, z=516, k=15

This cubic solution (91,185,516) is a special solution with a low k-value, the complete solution is given in Section (8.1.1).

7.3.58 n=3, x=92 to 96

There are no solutions for x in the range 92 to 96, and y in the range x+1 to 9999.

7.3.59 n=3, x=97

Prime x, $2mn+1$ form, i.e. x=(2.16.3+1), n=3, m=16.

n=3, x=97, y=3171, z=6124, k=105

7.3.60 n=3, x=98 and 102

There are no solutions for x in the range 98 to 102, and y in the range x+1 to 9999.

7.3.61 n=3, x=103

Prime x of $2mn+1$ form, x=(2.17.3+1), n=3, m=17.

Composite y has $2mn+1$ prime factor 67, i.e. x=8(2.11.3+1), n=3, m=11.

n=3, x=103, y=536, z=6223, k=701

7.3.62 n=3, x=104 to 108

There are no solutions for x in the range 104 to 108, and y in the range x+1 to 9999.

7.3.63 n=3, x=109

Prime x of $2mn+1$ form, x=(2.18.3+1), n=3, m=18.

Composite y = 3171 factors as 3.7.151, of which two are of the $2mn+1$ form, i.e. 7 and 151, where 151= (2.25.3+1), n=3, m=25, plus n also divides y.

Composite z = 30970 factors as 2.5.19.163, of which two are of the $2mn+1$ form, i.e. 19 and 162, where 151= (2.27.3+1), n=3, m=27.

n=3, x=109, y=3171, z=30970, k=2772

7.3.64 n=3, x=110

There are no solutions for x=110, and y in the range 111 to 9999.

7.3.65 n=3, x=111

n=3, x=111, y=6700, z=48511, k=3156

7.3.66 n=3, x=112 to 116

There are no solutions for x in the range 112 to 116, and y in the range x+1 to 9999.

7.3.67 n=3, x=117

n divides x twice

Composite x has $2mn+1$ factor 13, i.e. x=3.3(2.2.3+1), n=3, m=2.

n=3, x=117, y=323, z=1256, k=41

7.3.68 n=3, x=118 to 188

n=3, x=119, y=325, z=4269, k=471
n=3, x=126, y=2797, z=7141, k=136
n=3, x=133, y=632, z=1005, k=9 * low k-value
n=3, x=133, y=6364, z=29857, k=1043
n=3, x=139, y=868, z=5459, k=246
n=3, x=148, y=4171, z=16723, k=446
n=3, x=151, y=279, z=910, k=19 * low k-value
n=3, x=172, y=6669, z=23149, k=456
n=3, x=182, y=711, z=4559, k=160

7.3.69 n=3, x=189

n divides x three times

Composite x has $2mn+1$ factor 7, i.e. x=3.3.3(2.1.3+1), n=3, m=1.

There are no solutions (189,y,z) from y=190 to 9999 for x=189, n=3.

7.3.70 n=3, x=190 to 272

n=3, x=209, y=7891, z=18396, k=189
n=3, x=238, y=1769, z=10881, k=280
n=3, x=241, y=6331, z=38812, k=983
n=3, x=245, y=1371, z=6536, k=126

n=3, x=245, y=6571, z=32136, k=636
n=3, x=247, y=903, z=3775, k=63
n=3, x=266, y=1989, z=6437, k=76

7.3.71 n=3, x=273

n divides x

Composite x has two $2mn+1$ factors 7 and 13, x=3 (2.1.3+1) (2.2.3+1).

There are no solutions (273,y,z) from y=274 to 9999 for x=273, n=3.

7.3.72 n=3, x=274 to 350

There are no solutions for x in the range 274 to 350, and y in the range x+1 to 9999.

7.3.73 n=3, x=351

n divides x three times

Composite x has the $2mn+1$ factor 13, x=3.3.3. (2.2.3+1).

n	x	y	z	k
3	351	1393	8611	151
3	351	5449	27175	383
3	351	5450 to 9999	none	

7.3.74 n=3, x=352 to 495

There are no solutions for x in the range 352 to 495, and y in the range x+1 to 9999.

7.3.75 n=3, x=496

n=3, x=496, y=1317, z=8869, k=120

7.3.76 n=3, x=497 to 557

There are no solutions for x in the range 497 to 557, and y in the range x+1 to 9999.

7.3.77 n=3, x=558

n=3, x=558, y=3667, z=41275, k=832

7.3.78 n=3, x=559 to 675

There are no solutions for x in the range 559 to 675, and y in the range x+1 to 9999.

7.3.79 n=3, x=676

n=3, x=676, y=9397, z=6631, k=69

7.3.80 n=3, x=677 to 783

There are no solutions for x in the range 677 to 783, and y in the range x+1 to 9999.

7.3.81 n=3, x=784

n=3, x=784, y=3683, z=38803, k=521

7.3.82 n=3, x=785 to 947

There are no solutions for x in the range 785 to 947, and y in the range x+1 to 9999.

7.3.83 n=3, x=948 to 4655

n=3, x=948, y=4663, z=29419, k=195
n=3, x=1121, y=3744, z=33185, k=262
n=3, x=1261, y=1324, z=8305, k=41
n=3, x=1302, y=4693, z=23155, k=87
n=3, x=1327, y=2196, z=14911, k=76
n=3, x=1597, y=8321, z=71398, k=383
n=3, x=1736, y=2619, z=13715, k=41
n=3, x=1809, y=7736, z=27545, k=53
n=3, x=1838, y=2587, z=31985, k=215
n=3, x=2217, y=4223, z=40388, k=174
n=3, x=3708, y=4355, z=15323, k=14
n=3, x=4655, y=5836, z=93691, k=323

7.3.84 n=3, x=4656 to 9999

There are no solutions for n=3, x=4656 to 9999, y=x+1 to 9999.

7.3.85 n=3, x=10000 to 29999

n=3, x=11259, y=13244, z=151619, k=154
n=3, x=12871, y=17249, z=24440, k=269
n=3, x=14237, y=15678, z=292805, k=384

Checked to x=29999.

7.4 n=5

7.4.1 n=5, x=2

n	x	y	z	k

5	2	3	11 (x)	2436
5	2	123	155	1608 0000
5	2	151	279	1913 1952
5	2	393	2081	238 5401 2236
5	2	775 Note *	4697	3139 7670 5111
5	2	1321	9513	3 0996 6492 6960
5	2	1741	1991	22 0566 0239
5	2	4697	24257	36 8450 8459 2596
5	2	9223	56375	547 5113 3268 7976
5	2	4698 to 9999	none	

(x) denotes that the solution is an 'extended form' of the 'x plus y' solution, see Section (5.3.1).

Note *. The y-value of 775 is special because it factors as 775=5.5.31 hence n divides both 775 twice, and 31 is of the 2mn+1 form, i.e. 31=2.3.5+1, m=1. This means that it has 25 (5^2) unity roots, 24 of which are non-trivial. The value of 775 occurs in several other of the n=5 tables.

7.4.2 n=5, x=3

n	x	y	z	k
5	3	7	31 (x)	43951
5	3	44	191	1007 5757
5	3	61	88	27 5256
5	3	217	451	6191 2551
5	3	253	808	5 5987 9456
5	3	1661	6383	3327 2847 3991
5	3	2525	4898	732 1247 0176
5	3	2387	3062	87 4158 6951
5	3	2525	4898	732 1247 0176
5	3	3038	8921	6917 5366 6135
5	3	3056	7619	3637 3485 4740
5	3	3982	18505	9 8114 5520 3815
5	3	4009	30916	75 9556 2809 8387
5	3	4741	20542	12 5110 9155 7468
5	3	9913	53317	271 6689 2117 1967
5	3	9914 to 9999	none	

7.4.3 n=5, x=4

n	x	y	z	k
5	4	11	75	71 9056
5	4	13	121 (x)	412 2227
5	4	61	341	5540 4894
5	4	71	375	6961 4716
5	4	707	3063	311 0455 1733

5	4	781	3925	759 4726 3226
5	4	821	1025	2 2530 5915
5	4	7157	25113	13 8671 1534 4633
5	4	7929	61861	461 7153 4790 8203
5	4	8501	55365	276 2957 1107 8260
5	4	8502 to 9999	none	

7.4.4 n=5, x=5

n divides x

n	x	y	z	k
5	5	11	31	16695 *
5	5	51	311 (x)	3668 1735
5	5	88	213	462 1750
5	5	331	651	1 0483 6295
5	5	571	2976	274 6709 0130
5	5	941	2761	122 9431 3695
5	5	1111	2556	75 6429 0745
5	5	2173	2973	56 9040 7575
5	5	3124	9089	4348 0550 8115
5	5	3691	3976	42 0566 8310
5	5	7843	77768	932 7107 2397 3825
5	5	8528	63653	384 9812 3808 4250
5	5	9511	59911	270 8858 3490 7775
5	5	9512 to 9999	none	

Note *. This solution, (5,11,31), is also an MFLT quintic solution, see [1]#1 for the complete solution with unity roots.

(x) denotes that the solution is an ‘extended form’ of the ‘x plus y’ solution, see Section (5.3.1).

7.4.5 n=5, x=6

n	x	y	z	k
5	6	775	1381	7 3866 9775
5	6	6787	21601	5 3300 8865 7519
5	6	6788 to 9999	none	

7.4.6 n=5, x=7

n	x	y	z	k
5	7	11	123	297 2535
5	7	18	151 (x)	412 5976
5	7	93	205	266 0775

5	7	393	3550	577 3186 6760
5	7	4343	13079	9586 3531 2431
5	7	4413	15025	1 6461 7372 0231
5	7	8646	20077	2 6448 5333 2971
5	7	8647 to 9999	none	

7.4.7 n=5, x=8

n	x	y	z	k
5	8	297	1601	27 6454 3751
5	8	1189	1661	6 4980 8087
5	8	8371	27979	9 1289 1223 7615
5	8	8372 to 9999	none	

7.4.8 n=5, x=9

n	x	y	z	k
5	9	11	155	583 0295
5	9	31	382	7632 1644
5	9	2351	18209	5 2886 4533 2220
5	9	2352 to 9999	none	

7.4.9 n=5, x=10

n divides x

n	x	y	z	k
5	10	2911	9161	2411 6822 4375
5	10	2912 to 9999	none	

7.4.10 n=5, x=11

Prime x, $2mn+1$ form, 11 = 2.5 + 1, n=5, m=1

n	x	y	z	k
5	11	12	71	19 2469
5	11	41	403	5848 4247
5	11	53	488	9727 5656
5	11	284	1475	15 1475 4026
5	11	311	707	7183 1015
5	11	700	1271	3 2174 2229
5	11	909	1550	5 3721 4932
5	11	1312	6219	1036 0344 1002
5	11	1325	3191	70 2594 0887
5	11	2524	4555	146 9500 6100
5	11	3275	7526	876 6428 8943

5	11	4965	9641	1524 5894 0535
5	11	8494	37275	20 6489 8651 9176
5	11	9524	10055	231 8199 9035
5	11	9525 to 9999	none	

7.4.11 n=5, x=12

n	x	y	z	k
5	12	275	707	7503 7716
5	12	276 to 9999	none	

7.4.12 n=5, x=13

n	x	y	z	k
5	13	651	3121	112 0673 7959
5	13	1931	16244	2 7735 4992 4790
5	13	1932 to 9999	none	

7.4.13 n=5, x=14

n	x	y	z	k
5	14	843	4091	237 2471 0712
5	14	1831	8565	2098 4524 8915
5	14	1832 to 9999	none	

7.4.14 n=5, x=15

n divides x

n	x	y	z	k
5	15	31	151	1117615
5	15	4336	20191	2 5541 8519 3535
5	15	4337 to 9999	none	

7.4.15 n=5, x=16

n	x	y	z	k
5	16	1573	13237	1 2198 2991 1493
5	16	2379	20491	4 6315 7848 1724
5	16	2380 to 9999	none	

7.4.16 n=5, x=17

n	x	y	z	k
5	17	429	3761	274 3460 6415
5	17	9603	24461	2 1725 6132 8591
5	17	430 to 9999	none	

7.4.17 n=5, x=18

n	x	y	z	k
5	18	71	521	5765 0039
5	18	72 to 9999	none	

7.4.18 n=5, x=19

n	x	y	z	k
5	19	2251	9110	1608 9561 8835
5	19	2252 to 9999	none	

7.4.19 n=5, x=20

n divides x

n	x	y	z	k
5	20	5401	27061	4 9628 7833 0315
5	20	5402 to 9999	none	

7.4.20 n=5, x=21

n	x	y	z	k
5	21	251	671	3817 7239
5	21	252 to 9999	none	

7.4.21 n=5, x=22

Composite x has the $2mn+1$ factor 11, i.e. x=2(2.5+1), n=5, m=1.

n=5, x=22, y=393, z=2455, k=42 0092 6635

7.4.22 n=5, x=23

n=5, x=23, y=41, z=271, k=5719119

7.4.23 n=5, x=24

There are no solutions for x=24, y=25 to 9999

7.4.24 n=5, x=25

n divides x twice

n	x	y	z	k
5	25	279	844	7246 1588
5	25	541	3641	129 9313 6095

5	25	1604	5409	212 9739 5546
5	25	1859	15404	1 2114 4450 2749
5	25	5471	37556	0014 5439 4103 9742
5	25	5981	6461	0037 3189 5911
5	25	8168	9753	260 5441 1968
5	25	1860 to 2000	none	

7.4.25 n=5, x=26

n=5, x=26, y=175, z=1111, k=0003 3481 3316

7.4.26 n=5, x=27

There are no solutions for x=27, y=28 to 9999

7.4.27 n=5, x=28

n=5, x=28, y=3205, z=180731, k=1 1886 6302 7155

7.4.28 n=5, x=29

n=5, x=29, y=3183, z=9563, k=902 3274 6511

7.4.29 n=5, x=31

Composite *x* has the $2mn+1$ form i.e. x=(2.3.5+1), n=5, m=3.

n	x	y	z	k
5	31	71	723	1 2414 4967
5	31	77	213	85 6935
5	31	425	3091	69 2825 8031
5	31	764	4575	184 9494 0368
5	31	2009	18700	1 9634 4427 5266
5	31	2410	5231	98 1410 4600
5	31	5654	38581	12 6399 9975 9179
5	31	5655 to 9999	none	

7.4.30 n=5, x=32

n=5, x=32, y=993, z=4961, k=0190 5627 9916

7.4.31 n=5, x=33

Composite x has the $2mn+1$ factor 11, i.e. x=3(2.5+1), n=5, m=1.

There are no solutions (33,y,z) from y=34 to 3000 for x=33, n=5.

7.4.32 n=5, x=34 to 40

n=5, x=34, y=6483, z=26509, k=0002 2384 0336 8859
n=5, x=39, y=8921, z=85400, k=0152 8790 3553 3472
n=5, x=40, y=151, z=911, k=0001 1402 0295
n=5, x=40, y=713, z=3513, k=0053 3842 9150

7.4.33 n=5, x=41

Prime x is of the $2mn+1$ form, i.e. x=(2.4.5+1), n=5, m=4.

n	x	y	z	k
5	41	44	505	3605 1660
5	41	49	825	2 3058 7311
5	41	125	241	63 3423
5	41	132	941	1 4468 9529
5	41	220	2441	39 3606 2600
5	41	1519	14300	6714 2366 7266
5	41	1925	2141	1 0979 6135
5	41	2101	8857	713 8555 6415
5	41	2351	18538	1 2251 8248 0252
5	41	7810	44651	12 4113 4249 6355
5	41	8133	35981	5 0234 4853 9399
5	41	8134 to 3000	none	

7.4.34 n=5, x=44

Composite x has the $2mn+1$ factor 11, i.e. x=4(2.5+1), n=5, m=1.

n	x	y	z	k
5	44	71	575	3499 0226
5	44	1349	13097	4956 9855 0587
5	44	3813	4769	20 7574 9749
5	44	4555	22119	1 1938 7820 8325
5	44	4556 to 9999	none	

7.4.35 n=5, x=46

There are no solutions for n=5, x=46, y=47 to 9999

7.4.36 n=5, x=47

n	x	y	z	k
5	47	61	484	1913 9747
5	47	8118	37775	5 3342 3163 7312
5	47	8119 to 9999	none	

7.4.37 n=5, x=48

There are no solutions for n=5, x=48, y=49 to 9999

7.4.38 n=5, x=49

n=5, x=49, y=1936, z=5905, k=0127 6820 5910
n=5, x=49, y=8081, z=74525, k=0077 9004 9937 4951

7.4.39 n=5, x=50 to 52

There are no solutions for n=5, x=50 to 52, y=x+1 to 9999.

7.4.40 n=5, x=53

n=5, x=53, y=2651, z=20618, k=0001 2861 3122 4056

7.4.41 n=5, x=54

There are no solutions for n=5, x=54, y=55 to 9999.

7.4.42 n=5, x=55

n divides x

Composite *x*, factor $2mn+1$ form, 55 =5(2.5 + 1), n=5, m=1.

n	x	y	z	k
5	55	76	1271	6 2431 7520
5	55	131	426	455 8195
5	55	431	3131	40 5387 9575
5	55	432 to 9999	none	

7.4.43 n=5, x=56

There are no solutions for n=5, x=56, y=57 to 9999.

7.4.44 n=5, x=57 to 60

n=5, x=57, y=691, z=5764, k=0280 2413 5987
n=5, x=59, y=328, z=3691, k=0095 9064 7262

7.4.45 n=5, x=61

Prime x, $2mn+1$ form, 61=(2.6.5 + 1), n=5, m=6.

n	x	y	z	k
5	61	202	341	101 7084
5	61	3875	8261	192 5541 6023
5	61	6435	16441	1844 2801 6615
5	61	6454 to 9999	none	

7.4.46 n=5, x=62 to 65

There are no solutions for n=5, x=62 to 65, y=x+1 to 9999.

7.4.47 n=5, x=66

Composite x, factor $2mn+1$ form, 66 =6(2.5 + 1), n=5, m=1.

n=5, x=66, y=305, z=1451, k=2 2011 3935

7.4.48 n=5, x=67 to 69

There are no solutions for n=5, x=67 to 69, y=x+1 to 9999.

7.4.49 n=5, x=70

n=5, x=70, y=2101, z=15541, k=3966 1774 1360

7.4.50 n=5, x=71

Prime x, $2mn+1$ form, 71=(2.7.5 + 1), n=5, m=7.

n	x	y	z	k
5	71	341	764	13822793
5	71	3361	11166	649 8142 1644
5	71	7777	10004	129 8934 8762
5	71	7778 to 9999	none	

7.4.51 n=5, x=72

There are no solutions for n=5, x=72, y=73 to 9999.

7.4.52 n=5, x=73

n	x	y	z	k
5	73	2343	4168	16 6542 9366
5	73	6191	10498	0249 5767 3916
5	73	6192 to 9999	none	

7.4.53 n=5, x=74

There are no solutions for n=5, x=74, y=75 to 9999.

7.4.54 n=5, x=75

n divides x twice

n	x	y	z	k
5	75	2101	5041	40 4653 3111
5	75	8456	14411	632 7599 7952
5	75	8457 to 9999	none	

7.4.55 n=5, x=76 to 80

There are no solutions for n=5, x=76 to 80, y=x+1 to 9999.

7.4.56 n=5, x=81

n divides x twice

n=5, x=81, y=505, z=3421, k=33 4815 4215

7.4.57 n=5, x=82 to 109

n=5, x=82, y=2081, z=4697, k=0028 0361 5676
n=5, x=82, y=6151, z=8401, k=0077 9765 5499
n=5, x=93, y=211, z=484, k=2751740
n=5, x=93, y=275, z=488, k=2090926
n=5, x=100, y=5381, z=15921, k=1188 7726 2027
n=5, x=101, y=355, z=2706, k=0014 9535 5780

n=5, x=101, y=1012, z=1481, k=40055069
n=5, x=101, y=2799, z=18875, k=4489 4426 1391
n=5, x=101, y=5403, z=7376, k=0042 8016 9944
n=5, x=101, y=5747, z=12988, k=0481 9217 7810

7.4.58 n=5, x=102 to 104

There are no solutions for x=102 to 104, y=x+1 to 9999.

7.4.59 n=5, x=105

n=5, x=105, y=911, z=5951, k=0131 1039 0015

7.4.60 n=5, x=106 to 109

There are no solutions n=5, x=106 to 109, y=x+1 to 9999.

7.4.61 n=5, x=110

Composite *x*, factor $2mn+1$ form, 110 =10(2.5 + 1), n=5, m=1.

n=5, x=110, y=3441, z=7991, k=106 1327 0275

7.4.62 n=5, x=111 to 130

There are no solutions for n=5, x=111 to 130, y=x+1 to 9999.

7.4.63 n=5, x=131

Prime *x*, factor $2mn+1$ form, 131 =(2.13.5 + 1), n=5, m=13.

n=5, x=131, y=1804, z=6663, k=83 2796 9139

7.4.64 x=132 to 141

There are no solutions for n=5, x=132 to 141, y=x+1 to 9999.

7.4.65 n=5, x=142

Composite *x*, factor $2mn+1$ form, 142 =2(2.7.5 + 1), n=5, m=7.

n	x	y	z	k
5	142	869	4803	43 1179 4023
5	142	1023	2221	1 6403 3091
5	142	1024 to 9999	none	

7.4.66 n=5, x=143 to 150

There are no solutions for n=5, x=143 to 150, y=x+1 to 9999.

7.4.67 n=5, x=151

Prime x, $2mn+1$ form, 151 =(2.15.5 + 1), n=5, m=15.

n	x	y	z	k
5	151	225	961	25083423
5	151	1769	8184	167 8621 2944
5	151	1770 to 9999	none	

7.4.68 n=5, x=152 to 154

There are no solutions for x=152 to 154, y=x+1 to 9999.

7.4.69 n=5, x=155

Composite x, factor $2mn+1$ form, 155 =5(2.3.5 + 1), n=5, m=3.

n=5, x=155, y=3093, z=6248, k=0030 8421 8070
n=5, x=155, y=3157, z=6252, k=0030 1975 8180

7.4.70 n=5, x=156

There are no solutions for n=5, x=156, y=157 to 9999.

7.4.71 n=5, x=157

n=5, x=157, y=1891, z=12881, k=0927 2087 5719

7.4.72 =5, x=158 to 163

There are no solutions for n=5, x=158 to 163, y=x+1 to 9999.

7.4.73 =5, x=164 to 181

n=5, x=164, y=311, z=1375, k=70038834
n=5, x=165, y=1171, z=1681, k=34547095
n=5, x=181, y=1405, z=8866, k=0242 9475 9335

7.4.74 n=5, x=182

n=5, x=182, y=8283, z=74531, k=0020 4682 8149 5116

7.4.75 n=5, x=183 to 186

There are no solutions for n=5, x=183 to 186, y=x+1 to 9999.

7.4.76 =5, x=187

n=5, x=187, y=3550, z=10027, k=0151 4228 7896

7.4.77 n=5, x=188 to 190

There are no solutions for n=5, x=188 to 190, y=x+1 to 9999.

7.4.78 =5, x=191

n=5, x=191, y=465, z=1136, k=18533115
n=5, x=191, y=941, z=1364, k=16248383
n=5, x=191, y=2305, z=8866, k=0140 1814 2800
n=5, x=191, y=5425, z=11986, k=0195 4048 9352
n=5, x=191, y=5500, z=26691, k=4829 4974 6815

7.4.79 n=5, x=192 to 217

There are no solutions for n=5, x=192 to 217, y=x+1 to 9999.

7.4.80 n=5, x=217

n=5, x=217, y=1464, z=7561, k=0102 8484 9090

7.4.81 n=5, x=220

n=5, x=220, y=571, z=1051, k=9249320
n=5, x=220, y=9113, z=27273, k=2748 1147 9860

7.4.82 n=5, x=221 to 224

There are no solutions for n=5, x=221 to 224, y=x+1 to 9999.

7.4.83 n=5, x=225 to 280

n=5, x=251, y=541, z=2046, k=0001 2887 7684
n=5, x=264, y=271, z=655, k=2514160

7.4.84 n=5, x=281

n=5, x=281, y=925, z=6941, k=0089 2939 0631
n=5, x=281, y=1255, z=3641, k=0004 9592 1935
n=5, x=281, y=5704, z=45729, k=0002 7281 4762 3269

7.4.85 n=5, x=282, 283

There are no solutions from y=282 to 9999.

7.4.86 n=5, x=284

n=5, x=284, y=5821, z=43593, k=0002 1844 0244 1709

7.4.87 n=5, x=285 to 670

n=5, x=297, y=2351, z=20522, k=2540 1600 7036
n=5, x=303, y=8897, z=91856, k=0026 4082 8490 7831
n=5, x=305, y=837, z=2222, k=94759615
n=5, x=319, y=4965, z=15724, k=0384 7484 1635
n=5, x=331, y=2838, z=10651, k=0136 8161 7544
n=5, x=331, y=3245, z=12721, k=0243 5414 9735
n=5, x=341, y=1528, z=2221, k=39498329
n=5, x=355, y=573, z=1048, k=5613900
n=5, x=363, y=2542, z=10675, k=0140 6231 0812
n=5, x=385, y=1171, z=2501, k=84823215
n=5, x=385, y=2356, z=10721, k=0145 5735 6120
n=5, x=393, y=3131, z=17324, k=0731 8689 5715
n=5, x=393, y=3511, z=12751, k=0191 2782 6959
n=5, x=393, y=6100, z=55513, k=0003 9614 0685 0964
n=5, x=401, y=825, z=2501, k=0001 1779 0615
n=5, x=403, y=497, z=2265, k=0001 3131 4575
n=5, x=403, y=2525, z=3903, k=0002 0220 3175
n=5, x=422, y=1057, z=2697, k=0001 1750 6236
n=5, x=497, y=4863, z=6200, k=0004 2987 3946
n=5, x=541, y=843, z=1201, k=3700039
n=5, x=550, y=4371, z=27481, k=2372 1514 7851
n=5, x=550, y=5611, z=27371, k=1818 0358 4496
n=5, x=573, y=8107, z=52297, k=0001 6101 0215 4471
n=5, x=589, y=671, z=2020, k=41868515
n=5, x=631, y=3275, z=4431, k=0001 4538 1935
n=5, x=631, y=6050, z=10761, k=0033 1526 8423
n=5, x=656, y=1325, z=6221, k=0017 2236 4346

7.4.88 n=5, x=671

Prime x, $2mn+1$ form, 671=(2.67.5 + 1), n=5, m=67.

n	x	y	z	k
5	671	850	8831	106 6334 3648
5	671	851 to 9999	none	

7.4.89 n=5, x=672 to 6101

n=5, x=691, y=6369, z=41221, k=6559 7388 8239
n=5, x=715, y=2121, z=9661, k=0057 4141 7055
n=5, x=761, y=6603, z=35684, k=3226 0711 4665
n=5, x=775, y=1861, z=12401, k=0163 9627 8423
n=5, x=825, y=6481, z=7321, k=0002 4514 3471
n=5, x=843, y=3157, z=7444, k=0011 3792 9896
n=5, x=861, y=4499, z=48305, k=0001 4055 4953 7735
n=5, x=880, y=1681, z=5921, k=0008 2927 1115
n=5, x=915, y=3362, z=31067, k=3028 1125 3540
n=5, x=917, y=1891, z=4593, k=0002 5352 2935
n=5, x=968, y=1267, z=3075, k=71809066
n=5, x=991, y=1704, z=3535, k=89906110
n=5, x=991, y=7052, z=23523, k=0437 0507 4635
n=5, x=1055, y=6736, z=9471, k=0009 2615 1310
n=5, x=1111, y=1220, z=4231, k=0002 3566 1180
n=5, x=1171, y=2015, z=10986, k=0061 7206 2255
n=5, x=1181, y=7724, z=74117, k=0003 3080 6342 4134
n=5, x=1312, y=4309, z=50101, k=0001 1144 8236 0515
n=5, x=1320, y=2501, z=10781, k=0040 8925 9995
n=5, x=1321, y=2783, z=9348, k=0020 7213 1866
n=5, x=1353, y=2351, z=28706, k=2134 7085 7724
n=5, x=1383, y=1705, z=4168, k=0001 2600 5690
n=5, x=1500, y=6941, z=46841, k=4623 3873 9673
n=5, x=1519, y=8081, z=88000, k=0004 8854 5448 2706
n=5, x=1661, y=3690, z=3821, k=5026920
n=5, x=1804, y=2441, z=2525, k=-281761
n=5, x=1964, y=2761, z=3165, k=7453575
n=5, x=1965, y=6587, z=39842, k=1946 5274 5140
n=5, x=2068, y=7175, z=28983, k=0475 1121 1596
n=5, x=2096, y=9789, z=15005, k=0021 7857 4810
n=5, x=2253, y=3937, z=19525, k=0163 7886 8247
n=5, x=2281, y=8965, z=100081, k=0004 9060 1032 4975
n=5, x=2475, y=5921, z=31571, k=0677 7679 4295
n=5, x=2715, y=7336, z=17611, k=0047 6855 8550
n=5, x=3337, y=4691, z=78669, k=0002 4467 6631 0367
n=5, x=3421, y=6131, z=38652, k=1064 0389 4140
n=5, x=3439, y=7593, z=13768, k=0013 0452 1391
n=5, x=4405, y=8296, z=11341, k=0003 5386 2395
n=5, x=4653, y=8267, z=63257, k=4162 3239 9751
n=5, x=4869, y=6775, z=9124, k=0001 5356 5911
n=5, x=4972, y=5441, z=66461, k=7211 9885 0069
n=5, x=5011, y=5549, z=63723, k=5929 8300 4919
n=5, x=6101, y=7815, z=14981, k=0010 0376 6935

7.4.90 n=5, x=6102 to 9999

There are no solutions for x=6102 to 9999, y=x+1 to 9999.

7.4.91 n=5, x=10000 to 19999

n=5, x=10505, y=14793, z=208778, k=0012 2260 3689 4315
n=5, x=11261, y=18081, z=291737, k=0035 5767 4506 6015
n=5, x=11325, y=14743, z=71953, k=1604 6218 7431
n=5, x=11709, y=17507, z=247511, k=0018 3081 9838 4615
n=5, x=12401, y=14077, z=38553, k=0125 2939 7535
n=5, x=13725, y=13792, z=37597, k=0104 1680 9086
n=5, x=14701, y=18995, z=24851, k=0009 1051 7975
n=5, x=15557, y=19623, z=186755, k=0003 9846 5194 4895

7.5 n=7

7.5.1 n=7, x=2

n	x	y	z	k
7	2	113	145	339 4525 5984
7	2	565	4167	463 2998 8602 4269 2847
7	2	1363	3715	96 3470 1114 0872 5352
7	2	1479	1583	2013 7380 8073 0860
7	2	1943	7841	5979 9975 9428 2908 8171
7	2	3913	31761	1311 6711 7933 2934 6370 2716
7	2	4159	7239	1694 2798 6205 6136 2716
7	2	5783	12433	3 1784 9813 0530 0701 7299
7	2	6223	29465	525 7759 7281 9157 6847 4955
7	2	6223	46895	8545 3357 2660 5520 9448 1360
7	2	6224 to 9999	none	

7.5.2 n=7, x=3

n	x	y	z	k
7	3	10	43 (x)	2 1070 4348
7	3	31	127 (x)	451 1458 3655
7	3	145	1348	1 3792 7233 0296 3947
7	3	337	493	13 2109 7247 3919
7	3	701	1808	1 6587 4212 3355 5636
7	3	827	4118	196 5556 6612 1751 2099
7	3	1349	13208	13 1186 4883 2769 8886 2341
7	3	3175	24703	238 5794 0671 9261 8512 5623
7	3	3613	10756	1 4279 2677 2422 9773 6248
7	3	5597	14285	5 0534 6025 2729 4392 6095
7	3	7732	44863	3514 9148 4484 4383 9180 0469
7	3	8509	26803	145 1976 1971 4129 0721 2751

7	3	8510 to 9999	none	

(x) denotes that the solution is an ‘extended form’ of the ‘x plus y’ solution, see Section (5.3.1).

7.5.3 n=7, x=4

n	x	y	z	k
7	4	87	883	1362 0245 5854 2665
7	4	121	1093 (x)	3522 6988 8196 1261
7	4	1011	6235	1452 8016 3532 1976 2998
7	4	4757	17089	13 0873 1348 1204 0374 4581
7	4	4758 to 9999	none	

7.5.4 n=7, x=5

n	x	y	z	k
7	5	931	8916	1 0791 9671 5115 9032 4814
7	5	932 to 9999	none	

7.5.5 n=7, x=6

n	x	y	z	k
7	6	29	245 (x)	1 2429 3127 6476
7	6	449	1505	4312 5092 4582 3632
7	6	1207	11719	3 5767 1842 2923 8443 9720
7	6	4733	33173	469 2672 1621 1416 0795 1196
7	6	4744 to 9999	none	

(x) denotes that the solution is an ‘extended form’ of the ‘x plus y’ solution, see Section (5.3.1).

7.5.6 n=7, x=7

n divides x

n	x	y	z	k
7	7	15	127 (x)	399 6068 1551
7	7	113	1016	1390 5470 0100 1991
7	7	1514	11223	1 8855 1290 5151 2950 7000
7	7	1808	15367	10 4049 0084 6216 6572 8064
7	7	3282	18577	17 8901 7285 6979 1943 3779
7	7	4615	22822	43 7367 4471 9672 3390 7747
7	7	4616 to 9999	none	

7.5.7 n=7, x=8

n	x	y	z	k
7	8	87	215	1416 6067 5696
7	8	497	2017	1 6934 2146 7862 0274
7	8	5237	30421	189 1761 6172 3961 3526 6516
7	8	1809 to 9999	none	

7.5.8 n=7, x=9

n	x	y	z	k
7	9	23	239	9003 6173 0031
7	9	464	4649	241 7668 4565 1993 9974
7	9	2414	10991	8113 9581 7005 1352 9963
7	9	465 to 9999	none	

7.5.9 n=7, x=10

n	x	y	z	k
7	10	1877	7807	1206 2036 4335 1694 4471
7	10	1878 to 9999	none	

7.5.10 n=7, x=11

n	x	y	z	k
7	11	174	449	4494 2752 9118 1739
7	11	591	4859	202 4422 0421 3677 6663
7	11	1011	1583	1353 6274 0434 8495
7	11	2627	12406	1 2616 2304 2489 1552 4091
7	11	9299	88411	4 6688 3818 5157 7096 3254 9119
7	11	9300 to 2000	none	

7.5.11 n=7, x=12

n	x	y	z	k
7	12	1933	11281	8885 2696 3831 2024 1301
7	12	1934 to 9999	none	

7.5.12 n=7, x=13

There are no solutions (13,y,z) from y=14 to 9999 for x=13, n=7.

7.5.13 n=7, x=14

There are no solutions (14,y,z) from y=15 to 9999 for x=14, n=7

7.5.14 n=7, x=15

Although x is of the $2mn+1$ form, 15 is not prime – and the factors (3,5) are not of the $2mn+1$ form

There are no solutions (15,y,z) from y=16 to 9999 for x=15, n=7.

7.5.15 n=7, x=16

There are no solutions (16,y,z) from y=17 to 9999 for x=16, n=7.

7.5.16 n=7, x=17

n	x	y	z	k
7	17	674	2017	5873 8719 2354 4216
7	17	4733	45550	1110 0557 8689 4064 0131 9815
7	17	4734 to 9999	none	

7.5.17 n=7, x=18

There are no solutions (18,y,z) from y=19 to 9999 for x=18, n=7.

7.5.18 n=7, x=19

n	x	y	z	k
7	19	1143	2359	7885 6312 4479 2791
7	19	1491	7495	625 7390 0040 8816 8351
7	19	5741	9560	680 1407 6360 7504 7952
7	19	1492 to 9999	none	

7.5.19 n=7, x=20

There are no solutions (20,y,z) from y=21 to 9999 for x=20, n=7.

7.5.20 n=7, x=21

n divides x

n	x	y	z	k
7	21	43	127	46 4420 6455
7	21	239	1163	493 0084 5843 4751

7	21	240 to 9999	none	

7.5.21 n=7, x=22

n	x	y	z	k
7	22	29	381	4 7943 4373 5088
7	22	3955	7057	23 digits: 139 4901 6618 1856 6699
7	22	3956 to 9999	none	

7.5.22 n=7, x=23

There are no solutions for n=7, x=23, from y=24 to 9999.

7.5.23 n=7, x=24

n	x	y	z	k
7	24	71	551	16 4225 1799 3484
7	24	72 to 9999	none	

7.5.24 n=7, x=25 to 27

There are no solutions for n=7, x=25 to 27, from y=26 to 9999.

7.5.25 n=7, x=28

n	x	y	z	k
7	28	43	631	52 4263 5517 7483
7	28	1247	1695	599 9672 3330 2640
7	28	1248 to 9999	none	

7.5.26 n=7, x=29

Prime x, n, $2mn+1$ form, x=4.7+1, n=11, m=2

n	x	y	z	k
7	29	43	394	2 9999 2772 9036
7	29	337	3053	8 2857 6936 1044 2855
7	29	516	953	49 3791 9493 9241
7	29	672	4733	57 6834 5311 1526 8545
7	29	883	1356	230 7191 5356 2900
7	29	8199	31484	40 9586 1478 6586 6466 0249

7	29	8200 to 9999	none	

7.5.27 n=7, x=30 to 33

There are no solutions for n=7, x=30 to 33, from y=34 to 9999.

7.5.28 n=7, x=34

n	x	y	z	k
7	34	197	1421	1229 1896 5606 6748
7	34	72 to 9999	none	

7.5.29 n=7, x=35

n=7, x=35, y=3909, z=5729, k=0024 0634 8758 9385 7319
n=7, x=35, y=7547, z=55427, k=1097 7045 5518 4729 4166 2911

7.5.30 n=7, x=36

n=7, x=36, y=49, z=337, k=8303 8626 8216

7.5.31 n=7, x=37

n=7, x=37, y=43, z=635, k=0041 2071 1107 1311

7.5.32 n=7, x=38 to 42

There are no solutions for n=7, x=38 to 42, from y=39 to 9999.

7.5.33 n=7, x=43

Prime, x of the 2mn+1 form, i.e. 43=2.3.7+1, m=3.

n	x	y	z	k
7	43	638	659	6055 8015 8420
7	43	2229	7624	204 8516 6313 7261 7636
7	43	2667	5419	21 9268 0212 4723 3231
7	43	6125	26293	12 5444 0687 9526 1710 4655
7	29	2668 to 9999	none	

7.5.34 n=7, x=44 to 48

There are no solutions for n=7, x=44 to 48, from y=45 to 9999.

7.5.35 n=7, x=49

n	x	y	z	k
7	49	239	1723	2234 1753 4344 4823
7	49	2110	21189	8 7783 6575 1007 3892 6255
7	49	4999	10424	520 6984 1324 0418 9399
7	49	to 5000	none	

7.5.36 n=7, x=50 to 53

There are no solutions for n=7, x=50 to 53, from y=51 to 9999.

7.5.37 n=7, x=54

n=7, x=54, y=3319, z=27133, k=0022 2631 0316 2369 4436 3663

7.5.38 n=7, x=55,56

There are no solutions for n=7, x=55 to 56, from y=56 to 9999.

7.5.39 n=7, x=57

n=7, x=57, y=9065, z=82937, k=6298 6458 0234 1095 5818 8159

7.5.40 n=7, x=58 to 62

There are no solutions for n=7, x=58 to 62, from y=59 to 9999.

7.5.41 n=7, x=63

n=7, x=63, y=7673, z=64184, k=1446 2879 7712 6549 6399 4055

7.5.42 n=7, x=64 to 68

There are no solutions for n=7, x=64 to 68, from y=65 to 9999.

7.5.43 n=7, x=69

n=7, x=69, y=337, z=889, k=0021 2052 1953 7271

7.5.44 n=7, x=70

n=7, x=70, y=127, z=1247, k=0422 9576 7470 4432

7.5.45 n=7, x=71

n=7, x=71, y=281, z=1276, k=0216 3358 3694 4024
n=7, x=71, y=1009, z=1740, k=0378 8462 2812 1974
n=7, x=71, y=1921, z=4814, k=0009 1106 7392 3999 3828
n=7, x=71, y=5401, z=8816, k=0118 4669 1232 2516 7574
n=7, x=71, y=5881, z=21576, k=0002 4158 3642 2595 1234 8824

7.5.46 n=7, x=72 to 112

n=7, x=78, y=911, z=4733, k=0015 8198 2942 4386 8871
n=7, x=84, y=2633, z=25229, k=0011 6592 3395 1211 6458 6851
n=7, x=86, y=1769, z=4417, k=0004 8732 6181 3074 6396
n=7, x=98, y=127, z=1695, k=1905 4098 7598 1360
n=7, x=112, y=113, z=1905, k=3776 3601 9906 2640

7.5.47 n=7, x=113

n=7, x=113, y=172, z=1685, k=1177 5856 0476 5717
n=7, x=113, y=3397, z=14886, k=2834 5237 5674 3021 1471
n=7, x=113, y=5503, z=54576, k=0424 9444 9752 0611 0240 8323

7.5.48 n=7, x=114 to 9999

n=7, x=116, y=6287, z=58007, k=0522 3732 2162 0051 1725 6966
n=7, x=127, y=279, z=2359, k=4863 6193 7570 2031
n=7, x=127, y=1065, z=1822, k=0264 1752 0949 1956
n=7, x=127, y=2088, z=7687, k=0077 7964 3357 1700 5499
n=7, x=127, y=6833, z=21552, k=0001 1544 3759 0500 1001 4039
n=7, x=128, y=631, z=3319, k=0001 6550 0872 0998 8723
n=7, x=142, y=1933, z=12035, k=1107 0192 4699 2688 7715
n=7, x=142, y=7355, z=7947, k=0010 0901 0428 0091 9912
n=7, x=147, y=491, z=2339, k=2268 6876 2108 2495
n=7, x=174, y=4733, z=34349, k=0019 9433 6444 5701 2363 7656
n=7, x=181, y=381, z=3277, k=0001 7957 8761 6299 8623
n=7, x=196, y=1849, z=3557, k=5531 3756 6760 8901
n=7, x=196, y=6441, z=8233, k=0020 2433 8382 8342 1760
n=7, x=203, y=562, z=1899, k=0410 9897 6512 7536
n=7, x=209, y=339, z=3683, k=0003 5226 0789 1462 0263
n=7, x=211, y=956, z=10083, k=0520 9526 1892 1067 6740
n=7, x=213, y=5237, z=31610, k=0008 9430 0162 6435 5723 1815
n=7, x=239, y=441, z=4649, k=0009 5790 1285 8845 6239
n=7, x=239, y=7486, z=53135, k=0125 7870 4283 2654 9136 5520
n=7, x=239, y=441, z=4649, k=0009 5790 1285 8845 6239
n=7, x=245, y=2696, z=6581, k=0012 2750 5898 3772 8936
n=7, x=264, y=635, z=4859, k=0007 8506 1100 7248 8304
n=7, x=281, y=3479, z=13630, k=0655 8186 0153 3175 5864
n=7, x=284, y=337, z=989, k=0009 7707 4850 1371
n=7, x=284, y=3741, z=22085, k=0001 0921 3577 9942 3846 0330

n=7, x=306, y=565, z=5461, k=0015 3413 3413 7068 8304
n=7, x=357, y=659, z=4943, k=0006 1999 7163 3386 4855
n=7, x=381, y=2059, z=4372, k=8856 5019 1222 5366
n=7, x=381, y=2315, z=4376, k=7869 0498 7204 9821
n=7, x=425, y=547, z=4372, k=0003 0040 3106 3617 7753
n=7, x=463, y=473, z=1266, k=0018 7645 4135 3388
n=7, x=638, y=4789, z=13259, k=0177 6849 6968 8530 2171
n=7, x=645, y=6056, z=14981, k=0288 8899 5491 0735 4880
n=7, x=659, y=8091, z=8471, k=0001 9041 3619 6937 0159
n=7, x=701, y=8807, z=12209, k=0048 1934 7837 6743 8175
n=7, x=819, y=1247, z=14351, k=0855 3475 4713 0261 1143
n=7, x=910, y=6931, z=28771, k=8992 3251 4023 0820 6568
n=7, x=956, y=4691, z=11299, k=0046 3012 1513 4533 2308
n=7, x=1051, y=5894, z=13665, k=0104 8183 1438 5391 9591
n=7, x=1090, y=9773, z=66453, k=0080 8411 5465 7991 5439 3004
n=7, x=1134, y=3319, z=27133, k=0001 0601 4776 9400 4981 6083
n=7, x=1346, y=8385, z=81881, k=0267 0242 1675 1532 0678 4212
n=7, x=1379, y=5133, z=63506, k=0092 6729 5559 5890 1204 0200
n=7, x=1486, y=6035, z=53931, k=0027 4368 7594 1939 1524 3084
n=7, x=1533, y=2059, z=32719, k=0003 8868 8442 3173 0692 6151
n=7, x=1740, y=3107, z=33707, k=0002 7128 7685 0907 7736 2330
n=7, x=2758, y=3529, z=16871, k=0236 9136 4121 3543 3115
n=7, x=3381, y=6203, z=15863, k=0075 8662 6852 2446 1191
n=7, x=3479, y=5881, z=33930, k=7457 5376 8948 4402 4664

7.5.49 n=7, x=10000 to 19999

n=7, x=10224, y=14803, z=43891, k=4721 1790 6721 4523 1210
n=7, x=10558, y=15627, z=86665, k=22 digits : 0025 ... 6263
n=7, x=17141, y=18524, z=92029, k=22 digits : 0019 ... 8914

7.6 n=11

7.6.1 n=11, x=2

n	x	y	z	k
11	2	7	23 (x)	2 9590 3037 4488
11	2	89	299	23 digits : 320 8419 7061 5957 3530 6071
11	2	161	331	23 digits : 490 0805 2974 7996 5583 4571
11	2	171	1541 (x)	30 digits : 22 0800 6162 2384 4011 5720 2342 7251
11	2	481	3369 (x)	33 digits :

				1 9581 0076 7106 9245 1437 7246 3071 7580
11	2	1283	3851 (x)	33 digits : 2 7956 0685 3878 9077 6089 0726 5930 0396
11	2	1689	15203 (x)	39 digits
11	2	3473	23105	40 digits
11	2	4183	26745	41 digits
11	2	4623	39359	42 digits
11	2	4624 to 9999	none	

(x) denotes that the solution is an 'extended form' of the 'x plus y' solution, see Section (5.3.1).

7.6.2 n=11, x=3

n	x	y	z	k
11	3	5	23 (x)	2 7617 6727 2199
11	3	92	335	23 digits : 644 9687 8643 4848 8645 5927
11	3	160	1123 (x)	28 digits : 6645 9386 0599 6298 3883 9543 1917
11	3	170	683 (x)	26 digits : 43 3148 4474 3791 1463 7365 1244
11	3	397	445	24 digits : 1828 2601 4496 9442 2846 2495
11	3	511	2047 (x)	30 digits : 84 2628 5500 9208 2434 4988 3686 2295
11	3	3352	13915	38 digits
11	3	3565	24028	40 digits
11	3	3917	18032	39 digits
11	3	6352	13795	38 digits : 13 ... 5314
11	3	7262	33419	41 digits : 7 ... 8676
11	3	512 to 9999	none	

7.6.3 n=11, x=4

n	x	y	z	k
11	4	5	69 (x)	18 digits : 12 2309 7030 3269 9568
11	4	7	67 (x)	17 digits :

				6 5101 3501 6150 6661
11	4	73	661 (x)	26 digits : 54 5295 2861 3292 1775 6759 6465
11	4	89	713	26 digits
11	4	1985	2049	29 digits
11	4	2049	2113	29 digits
11	4	5501	24589	40 digits : 3672 ... 3964
11	4	5829	38893	42 digits : 33 .. 9598
11	4	9131	9255	35 digits : 174 ... 6197
11	44	to 9999	none	

7.6.4 n=11, x=5

n	x	y	z	k
11	5	69	89	17 digits : 8 4884 7315 9360 8959
11	5	178	1013	28 digits : 1278 5109 2897 3501 2188 4032 2792
11	5	706	1791	29 digits : 9 6197 9115 8557 3213 9450 0046 8547
11	5	2047	12477	37 digits
11	5	8724	83549	45 digits : 3 7995 0395 1757 9131 9100 3733 5893 4889 7883 1545 2490
11	5	9154	29749	41 digits : 1 1861 5593 8398 5685 4824 7095 4256 6379 2287 7704
11	5	2048 to 9999	none	

7.6.5 n=11, x=6

n	x	y	z	k
11	6	2317	6733	35-digits: 0137 7226 5288 4642 8980 6172 7607 4024 5408
11	6	8107	40321	42 digits : 23 3507 8645 1741 7994 1424 2429 8702 2758 1090 8071
11	6	2318 to 9999	none	

7.6.6 n=11, x=7

n	x	y	z	k
11	7	255	2047 (x)	30 digits: 72 3669 3950 6710 8170 0028 2563 1503
11	7	256 to 9999	none	

(x) denotes that the solution is an 'extended form' of the 'x plus y' solution, see Section (5.3.1).

7.6.7 n=11, x=8

n	x	y	z	k
11	8	427	3851 (x)	33 digits: 2 0999 9047 1629 9681 1880 5362 3243 6196
11	8	1985	2113	29 digits : 5 5539 1001 1427 4546 5163 4103 1088
11	8	3697	5681	34 digits : 11 7338 7168 3547 4567 2045 6574 5838 0456
11	8	6537	19481	39 digits : 150 5363 4035 6982 9405 2262 5533 7383 5026 7526
11	8	3698 to 9999	none	

7.6.8 n=11, x=9

n	x	y	z	k
11	9	89	161	20 digits : 1458 7674 1203 9892 1983
11	9	2164	21649 (x)	40 digits : 1161 1240 9788 5679 4875 7239 4955 4543 4367 6974
11	9	3631	16744	38 digits : 53 0057 9752 5978 6371 8189 7289 5954 9293 1329
11	9	3632 to 9999	none	

7.6.9 n=11, x=10

n	x	y	z	k

11	10	161	801	26 digits : 67 5303 9273 0482 4425 4987 8464
11	10	1407	2047	29 digits : 9 0323 4732 5034 1678 6308 4119 6224
11	10	8537	10787	36 digits : 2308 0146 1191 8906 2351 1296 9153 4336 6975
11	10	1408 to 9999	none	

7.6.10 n=11, x=11

n	x	y	z	k
11	11	276	419	23 digits : 543 7712 6520 3105 4559 9098
11	11	1655	9751	36 digits : 4268 7515 5914 4743 0005 7587 8210 2068 6583
11	11	2278	8273	35 digits : 0599 3541 8031 0161 0755 2160 3282 1510 7791
11	11	8879	53935	43 digits : 213 2799 7860 0689 3162 9412 4441 4839 4342 7757 2039
11	11	2279 to 9999	none	

7.6.11 n=11, x=12

n	x	y	z	k
11	12	4735	41407	42 digits : 26 0757 2751 2083 2217 5152 3864 9359 1523 7220 0272
11	12	4736 to 9999	none	

7.6.12 n=11, x=13

n	x	y	z	k
11	13	67	184	Note * 20 digits : 5106 8557 9354 3958 8226

11	13	67	353	Note * 23 digits : 0344 9283 5233 8733 3407 0279
11	13	356	3177	32 digits : 2263 4911 3533 0447 4993 7336 2621 4405
11	13	357 to 5000	none	

Note *. The first two solutions have the same *x* and *y* values, 13 and 67 respectively. They have both been verified as correct, with a complete solution in unity roots obtained as a check.

7.6.13 n=11, x=14

There are no solutions (14,y,z) from y=15 to 5000 for x=14, n=11.

7.6.14 n=11, x=15

There are no solutions (15,y,z) from y=16 to 5000 for x=15, n=11.

7.6.15 n=11, x=16

n	*x*	*y*	*z*	k
11	16	89	345	23 digits : 167 7570 6341 0124 0430 3024
11	16	3191	20263	39 digits : 0228 5549 6326 7316 6902 6930 0358 4177 3800 7385
11	16	4423	8119	35 digits : 0175 6481 0495 2418 6083 3876 1556 8167 9853
11	16	4424 to 9999	none	

7.6.16 n=11, x=17

There are no solutions (17,y,z) from y=18 to 9999 for x=17, n=11.

7.6.17 n=11, x=18

n	*x*	*y*	*z*	k
11	18	3917	18101	38 digits : 53 5549 4602 5973 4429 7557 5814 6549 3424 1756
11	18	3918 to 9999	none	

7.6.18 n=11, x=19

n=11, x=19, y=5812, z=7123
k=27 1977 2982 0108 4804 1536 2978 4603 6880

7.6.19 n=11, x=20

There are no solutions (20,y,z) from y=21 to 5000 for x=20, n=11.

7.6.20 n=11, x=21

There are no solutions (21,y,z) from y=22 to 5000 for x=20, n=11.

7.6.21 n=11, x=22

There are no solutions (22,y,z) from y=23 to 5000 for x=22, n=11.

7.6.22 n=11, x=23

Prime x, $2mn+1$ form, x=2.11+1, n=11, m=1

n	x	y	z	k
11	23	33	89	4 1081 8726 4216 2495
11	23	57	536	25 digits : 1 4929 4980 0974 1130 8146 3546
11	23	61	267	22 digits : 13 1236 5799 8343 0534 5295
11	23	75	623	25 digits : 5 1061 2411 6398 5664 8544 0503
11	23	165	683	25 digits : 5 8209 6764 7929 3303 0525 1359
11	23	363	947	26 digits : 69 4790 5914 0663 1014 7451 6263
11	23	585	5288	34 digits 12 7066 4107 7962 4719 8743 0587 7289 2724
11	23	726	1277	689 2435 3851 4185 0464 4433 7095
11	23	1257	8576	35 digits : 744 3627 1195 6444 5223 5641 8199 5236 4346
11	23	1276	3851	32 digits : 2444 2922 8399 9768 5657 1788 1156 8001

11	23	1709	15404	38 digits
11	23	2577	3980	32 digits : 1668 5191 7032 6232 3536 3052 3486 6170
11	23	3177	5360	33 digits
11	23	4492	13535	37 digits : 1 9971 5917 9801 3250 2507 2186 1109 5469 9234
11	23	9839	88574	45 digits : 1 3133 5999 0093 5227 2869 0707 1968 2760 9632 2911 5811
11	23	9840 to 9999	none	

7.6.23 n=11, x=26

n	x	y	z	k
11	26	267	683	25 digits : 3 1820 5935 6316 8867 7282 3728
11	26	268 to 9999	none	

7.6.24 n=11, x=27

n	x	y	z	k
11	27	397	1234	27 digits : 763 ... 0348
11	27	1235 to 9999	none	

7.6.25 n=11, x=28

n	x	y	z	k
11	28	2707	17183	38 digits : 29 ... 6439
11	28	2708 to 9999	none	

7.6.26 n=11, x=29

n	x	y	z	k
11	29	1068	4577	33 digits : 1 3026 8623 8141 4006 9349 2966 5087 5473
11	29	1069 to 9999	none	

7.6.27 n=11, x=31

n	x	y	z	k
11	31	230	1191	27 digits : 805 ... 9972
11	31	1340	3851	32 digits : 1726 ... 1463
11	31	5281	24656	39 digits : 507 1639 0080 4432 2402 4398 9512 4503 0001 3684
11	31	2708 to 9999	none	

7.6.28 n=11, x=35

n=11, x=35, y=6749, z=15709
k=3 8738 0671 8761 3567 3755 6116 9021 9984 0351

n=11, x=35, y=8779, z=12279
k=2472 5379 6146 5778 7174 3656 2707 4452 8975

7.6.29 n=11, x=43

n	x	y	z	k
11	43	368	3851	32 digits : 4533 ... 2038
11	43	2985	8188	35 digits : 105 ... 6691
11	43	2986 to 9999	none	

7.6.30 n=11, x=46

n=11, x=46, y=1191, z=4951
k=0001 6153 2383 8646 0932 4283 8939 0052 7224

n=11, x=46, y=2333, z=2613
k=0009 8525 9117 9549 4115 6009 9611 0236

n=11, x=46, y=7283, z=70953
k=0965 2492 8021 0655 0168 6094 6492 8631 8494 7680 2371

7.6.31 n=11, x=52 to 53

n=11, x=52, y=3323, z=15075
k=0003 5078 3774 8292 4861 8280 5249 6992 3246 0470

n=11, x=53, y=184, z=661
k=0001 6327 5327 8653 9939 8677 6540

7.6.32 n=11, x=59

n=11, x=59, y=92, z=859
k=0040 2987 7079 0428 9662 3145 6296

n=11, x=59, y=1541, z=14816
k=0005 6060 2516 5895 7006 6096 9675 2819 2231 7754

7.6.33 n=11, x=66

n=11, x=66, y=67, z=661
k=0003 6007 7393 8311 7544 2249 6311

7.6.34 n=11, x=67

n=11, x=67, y=242, z=1453
k=2586 8185 8878 5912 7245 7934 4643

n=11, x=67, y=605, z=727
k=8825 7294 2601 3244 5475 0007

n=11, x=67, y=1725, z=4942
k=7518 9888 2038 7637 9587 2107 7934 5280

n=11, x=67, y=3371, z=30406
k=2990 6070 7521 8583 3270 7998 7228 6472 6880 7331

n=11, x=67, y=4098, z=24493
k=0282 9912 8875 3095 7092 1154 9693 9628 5298 6219

n=11, x=67, y=9834, z=88573
k=4510 3322 9906 3372 0319 9428 6562 4310 2833 4528 9815

7.6.35 n=11, x=77

n=11, x=77, y=5028, z=18041
k=0009 4345 1824 6474 0473 6237 2237 9757 2178 9201

7.6.36 n=11, x=89

n=11, x=89, y=490, z=2049
k=0002 9911 1722 4933 7796 7993 0595 7804

n=11, x=89, y=706, z=3635
k=0640 9873 1913 3548 8637 4983 2732 2707

n=11, x=89, y=1412, z=3781
k=0475 1530 4169 8931 4018 4624 8279 6713

n=11, x=89, y=2662, z=21143
k=0075 3473 5997 6450 9364 4795 5726 5329 6260 8795

n=11, x=89, y=2806, z=12655
k=4218 3614 9327 3151 1152 8312 1100 5350 4239

n=11, x=89, y=3905, z=15709
k=0002 6331 4184 5425 5736 7932 0258 6529 9762 5519

7.6.37 n=11, x=92

n=11, x=92, y=859, z=7131
k=0043 0252 4710 4694 0109 3837 1768 3922 5248

n=11, x=92, y=7283, z=38615
k=0001 1001 7621 0416 2459 4460 9999 6789 7335 8002 8325

7.6.38 n=11, x=93 to 104

n=11, x=93, y=938, z=2333
k=0005 4757 3380 5354 6767 1617 0026 9984

n=11, x=95, y=3241, z=4666
k=1559 8618 7336 5717 7240 8992 9150 1340

n=11, x=104, y=267, z=683
k=7955 1483 8267 5108 9319 9020

7.6.39 n=11, x=115

n=11, x=115, y=993, z=1588
k=0887 9088 3454 8443 3868 7057 9363

n=11, x=115, y=2181, z=21976
k=0104 7449 1729 6027 1468 0471 9667 9291 8517 5023

n=11, x=115, y=6749, z=16214
k=0001 6178 4232 4743 8200 5334 1400 7641 9903 4216

7.6.40 n=11, x=121

n=11, x=121, y=804, z=4621
k=4563 7123 6154 3440 2968 7189 6125 5839

n=11, x=121, y=995, z=6231
k=0007 3277 2328 3706 2549 5772 4818 5266 9183

n=11, x=121, y=8900, z=16721
k=0001 5849 8445 0955 6534 5487 6732 9065 2035 6274

7.6.41 n=11, x=133

n=11, x=133, y=8349, z=21649
k=0020 3647 9833 2964 0666 5818 9247 6704 0521 1151

7.6.42 n=11, x=134

n=11, x=134, y=413, z=3851
k=1296 2248 2945 2596 2764 6586 5501 3315

n=11, x=134, y=5785, z=17359
k=0003 2050 6192 1824 5256 2479 6166 6992 0883 0935

7.6.43 n=11, x=137 to 4065

n=11, x=137, y=6482, z=8537
k=0022 0342 4632 2441 5955 2871 7312 0005 0104

n=11, x=139, y=1191, z=3415
k=0130 3096 2244 1066 5085 6419 7582 5047

n=11, x=142, y=4781, z=13869
k=3878 2950 4529 7510 5952 4988 5366 6556 8960

n=11, x=155, y=1453, z=12148
k=3107 7818 0678 4119 9792 7010 2245 1999 9614

n=11, x=160, y=2369, z=19009
k=0016 2520 3179 9428 3347 6303 7874 6071 9061 1064

n=11, x=166, y=419, z=2577
k=0018 5704 3485 7808 3338 9592 6062 6611

n=11, x=207, y=353, z=938
k=0007 2156 2565 3098 2152 4495 3064

n=11, x=230, y=1943, z=17543
k=0006 1778 5907 3262 5992 5276 3760 1852 6038 8920

n=11, x=242, y=1541, z=7349
k=0012 3215 2476 1328 5934 0917 7869 1534 9000

n=11, x=253, y=6707, z=38310
k=4013 0326 2313 9905 6733 8841 6537 3656 8953 0456

n=11, x=257, y=3851, z=6164
k=7955 2631 4595 3989 3147 5289 4921 4765

n=11, x=267, y=7613, z=15404
k=3698 9746 9628 4268 5952 4447 9607 9671 5826

n=11, x=269, y=661, z=3082
k=0043 4883 1543 6132 7405 6593 2689 2451

n=11, x=281, y=1322, z=3851
k=0193 1052 2391 6498 9169 0407 2088 4331

n=11, x=299, y=3026, z=11723
k=0541 8104 6690 3314 0052 5700 2788 4361 7776

n=11, x=331, y=5829, z=57316
k=0019 8306 3819 1956 1145 5180 4401 8434 9348 5910 0684

n=11, x=335, y=712, z=1607
k=0481 4795 6420 3794 8654 4946 5277

n=11, x=341, y=683, z=2047
k=5546 2681 7206 4887 1461 9140 7095

n=11, x=353, y=759, z=947
k=0001 9752 8623 6804 3104 7005 9063

n=11, x=359, y=397, z=6141
k=0005 3518 7570 3828 5094 4303 6549 0348 9303

n=11, x=398, y=9313, z=38623
k=1992 9082 1097 2349 5492 0848 4488 6967 1162 7319

n=11, x=419, y=1081, z=6999
k=0006 2275 9294 1007 2183 5839 8969 8735 2959

n=11, x=419, y=1219, z=7839
k=0017 1549 2861 9533 7787 9387 9350 6372 1919

n=11, x=445, y=1123, z=1288
k=0019 5771 6752 2057 3200 2949 4892

n=11, x=460, y=4357, z=17117
k=0001 0772 8593 2719 1566 3392 9975 1036 3545 1686

n=11, x=477, y=3436, z=9637
k=0042 1543 0321 4811 3151 7915 7806 1307 5986

n=11, x=528, y=683, z=9131
k=0111 7194 5324 3599 5178 2419 0534 2148 6768

n=11, x=529, y=4098, z=5089
k=0487 7819 3084 5322 8584 0461 8549 8616

n=11, x=547, y=2759, z=20263
k=0007 7321 1700 6025 4212 3032 0513 1230 2965 0375

n=11, x=597, y=683, z=10235
k=0309 3737 3121 1779 0333 5199 8008 0678 7903

n=11, x=617, y=726, z=4577
k=0900 7148 8267 4468 1275 9511 2547 0396

n=11, x=661, y=9768, z=88573
k=0460 2633 9434 1612 1890 6485 7385 2159 1069 0395 6771

n=11, x=708, y=3383, z=16127
k=4968 2173 7112 7287 8866 1315 7788 4105 5238

n=11, x=736, y=7459, z=36355
k=0734 6543 5349 7720 5990 1667 5373 8715 8089 9099

n=11, x=926, y=1027, z=16331
k=0001 4188 8225 6590 2707 8780 9536 8672 6185 4836

n=11, x=991, y=7866, z=55369
k=0003 4740 7510 1557 1550 8227 2676 2859 2858 1490 0583

n=11, x=1001, y=5028, z=18041
k=7257 3217 2805 6848 1608 8164 6992 4273 5916

n=11, x=1133, y=5028, z=19757
k=0001 5906 8686 0355 8593 5785 0523 5121 6420 8528

n=11, x=1157, y=9154, z=77573
k=0074 5009 1495 5629 6671 8444 3868 8605 6687 9432 0320

n=11, x=1191, y=1817, z=10592
k=0082 1321 1952 6102 8894 6665 7125 6848 9566

n=11, x=1389, y=6566, z=9131
k=0004 3001 0562 3731 3497 5955 7106 7644 8379

n=11, x=1453, y=1518, z=8053
k=0005 2004 3410 2460 2672 3438 5429 4093 0964

n=11, x=1453, y=8470, z=86923
k=0200 0774 0400 9520 3046 9648 9135 8052 4143 3685 9731

n=11, x=1505, y=4623, z=42248
k=2603 7612 0216 0451 2063 2814 6712 2219 4147 6650

n=11, x=1580, y=2113, z=27393
k=0071 2589 4544 8414 5269 6020 2828 3370 1738 5136

n=11, x=1757, y=4666, z=31021
k=0100 6564 1092 8815 2880 9902 3913 6210 4031 9256

n=11, x=1787, y=2113, z=30705
k=0197 2726 9044 9318 5490 2417 4992 7721 6292 2855

n=11, x=1853, y=2047, z=31695
k=0269 7207 1302 8387 5881 5228 4447 9004 7111 4295

n=11, x=1876, y=7197, z=58213
k=0003 3099 0458 8688 2029 3235 4406 5950 8625 1186 9678

n=11, x=2077, y=2179, z=37891
k=1347 9287 7416 0631 9316 2960 8949 2576 3720 2463

n=11, x=2189, y=9043, z=57168
k=0001 8835 2632 9452 1039 8038 4541 3541 5880 0751 7826

n=11, x=2267, y=2577, z=26756
k=0032 1844 3830 9855 7753 9221 8305 1361 5171 2425

n=11, x=2377, y=2783, z=4236
k=0027 7944 8411 4145 3207 4371 9897 7146

n=11, x=2577, y=2767, z=38548
k=1016 0028 4820 5598 6169 4969 1698 0658 8312 3528

n=11, x=2613, y=4405, z=30613
k=0062 8020 1701 4065 8969 6306 8772 5797 9755 4975

n=11, x=2648, y=5949, z=19757
k=5752 3757 5944 2281 8639 0391 0775 5409 0878

n=11, x=2747, y=5296, z=27723
k=0018 4329 8239 9736 0125 7682 3513 3318 0845 6953

n=11, x=2824, y=3415, z=48599
k=7621 1047 3903 4312 3081 3482 0471 4979 9699 5760

n=11, x=3015, y=7363, z=50338
k=4705 6264 3546 8300 4473 9070 7373 4672 5755 0315

n=11, x=3241, y=6738, z=11617
k=0020 4476 1908 2098 0849 4194 5258 4962 9780

n=11, x=3403, y=6339, z=60787
k=0003 1931 8736 6694 4369 1793 4511 9463 4079 9593 0863

n=11, x=3422, y=9637, z=54123
k=6540 2860 8524 8188 1118 5457 2050 3886 5842 3763

n=11, x=3427, y=5955, z=60787
k=0003 3752 9109 0341 9428 2547 8786 5452 9843 8774 5887

n=11, x=3436, y=3899, z=54123
k=0001 6099 4938 2981 4865 0827 7821 2629 8948 1073 5436

n=11, x=3551, y=8188, z=35907
k=0122 5357 1042 8220 2763 7718 5839 4080 2319 0605

n=11, x=3567, y=8537, z=62042
k=0002 7749 1383 9550 0594 2575 4135 9433 0887 3258 3484

n=11, x=3704, y=6073, z=11353
k=0015 7973 5190 4883 9792 9788 9158 3374 6916

n=11, x=3851, y=6329, z=60812
k=0002 8378 1613 7333 4031 0866 9308 1103 7137 3468 0521

n=11, x=4065, y=7702, z=44287
k=0927 0401 7548 6594 8522 1099 1746 2798 0894 4499

7.6.44 n=11, x=7164

n=11, x=7164, y=8119, z=12151
k=0011 8841 1906 8941 4979 0895 0452 0399 6848

7.6.45 n=11, x=8107 to 9999

n=11, x=8107, y=8165, z=156852
k=0001 3617 0880 2679 3193 1326 5658 1714 8876 4983 5178 1628

n=11, x=8273, y=9637, z=119997
k=0776 4240 9083 7143 3830 2813 7926 2311 2446 9070 8279

7.6.46 n=11, x=10000 to 19999

n=11, x=11585, y=12058, z=49193, k=39 digits : 0594 ... 1436
n=11, x=13077, y=13378, z=180895, k=45 digits : 0002 ... 8299

n=11, x=13731, y=19192, z=283459, k=47 digits : 0127 ... 0740

7.7 n=13

7.7.1 n=13, x=2

n	x	y	z	k
13	2	79	471	30 digits : 75 4386 3099 7341 0958 7745 3840 5460
13	2	395	2237	38 digits
13	2	901	6751	43 digits
13	2	4685	25247	49 digits
13	2	902 to 9999	none	

7.7.2 n=13, x=3

n	x	y	z	k
13	3	5	53 (x)	20 digits : 3275 0593 6171 1353 9495
13	3	316	583	31 digits : 162 5784 3119 0205 7271 8485 4812 6191
13	3	682	2731 (x)	38 digits
13	3	5777	6761	42 digits
13	3	6413	31826	50 digits
13	3	7853	19568	48 digits
13	3	683 to 9999	none	

(x) denotes that the solution is an ‘extended form’ of the ‘x plus y’ solution, see Section (5.3.1).

7.7.3 n=13, x=4

n=13, x=4, y=53, z=237
k=0148 1304 3209 5869 6111 4713 4390

n=13, x=4, y=2939, z=17331
k=0624 6315 1134 6930 1751 2491 3387 5302 4008 3857 5780 5878

n=13, x=4, y=4187, z=27147
k=0009 5653 1249 8030 9158 4893 6496 3795 7920 6511 8092 3589 1036

n=13, x=4, y=4889, z=12349
k=0006 4313 0208 8427 6342 2814 8863 9689 5162 6276 7842 4089

n=13, x=4, y=9987, z=53059
k=0001 2462 7443 3450 3779 2640 7401 8617 8804 0936 1144 3822 3672 3024

7.7.4 n=13, x=5

n=13, x=5, y=424, z=869
k=8747 0756 3318 0797 6349 2838 3488 5932

n=13, x=5, y=4376, z=17861
k=0481 7502 2152 0359 3567 0579 1417 3699 8496 4449 6180 4936

n=13, x=5, y=8374, z=68859
k=0027 1409 8318 3449 8522 9117 4938 2419 7524 5195 0529 2791 0953 5911

7.7.5 n=13, x=6

There are no solutions (6,y,z) from y=7 to 2000 for x=6, n=13.

7.7.6 n=13, x=7

n	x	y	z	k
13	7	9	79 (x)	9 3796 0492 5657 2621 3039
13	7	159	859	33 digits
13	7	316	2983	39 digits
13	7	1023	8191 (x)	44 digits
13	7	1024 to 9999	none	

7.7.7 n=13, x=8

n=13, x=8, y=79, z=159
k=4130 4709 3993 5833 8392 1954

n=13, x=8, y=2735, z=11183
k=0001 7484 4741 8751 3316 6877 3578 0603 3599 2574 9095 0560

n=13, x=8, y=4555, z=6123
k=0007 4578 0583 0715 1231 1135 2934 4256 1878 7400 8916

n=13, x=8, y=8065, z=8193
k=0026 2454 1749 7923 2202 3760 1955 1447 8363 5401 7488

n=13, x=8, y=8193, z=8321
k=0030 6821 6937 7481 2218 8277 4406 3329 7663 9993 6720

7.7.8 n=13, x=9

n=13, x=9, y=1847, z=5024
k=0001 5555 5435 3558 7062 5959 3931 4624 3054 8411 4234

n=13, x=9, y=5204, z=8957
k=0568 8498 3281 3618 9521 7692 7961 5772 4179 9417 0057

n=13, x=9, y=8743, z=43213
k=0538 8526 5030 8974 3700 0786 8970 2786 0384 9908 2289 7464 8351

7.7.9 n=13, x=10

There are no solutions (10,y,z) from y=11 to 5000 for x=10, n=13.

7.7.10 n=13, x=11

n=13, x=11, y=5371, z=17559
k=0145 3949 8244 1714 6153 5011 5234 4677 3435 7165 3387 0303

n=13, x=11, y=8269, z=14220
k=0007 5088 2118 1612 3725 0678 2911 9815 5803 7973 2442 4362

7.7.11 n=13, x=12

n=13, x=12, y=1565, z=6557
k=0033 6330 7784 8478 1048 5206 6119 3469 8829 8688 3616

7.7.12 n=13, x=13 to 15

There are no solutions for x=13 to 15, y=x+1 to 9999.

7.7.13 n=13, x=16

n=13, x=16, y=8065, z=8321
k=0028 5054 0896 0677 2156 5246 0000 3901 0888 6125 7936

n=13, x=16, y=8957, z=68253
k=0007 1314 5892 3472 5150 8411 1306 5336 9095 4658 5929 3100 1065 8370

7.7.14 n=13, x=17

There are no solutions for x=17, y=18 to 9999.

7.7.15 n=13, x=18

n=13, x=18, y=1027, z=4555
k=4315 3658 9864 4267 1850 9987 9459 5294 1998 1428

7.7.16 n=13, x=19

There are no solutions for x=19, y=20 to 9999.

7.7.17 n=13, x=20 to 26

n=13, x=20, y=8971, z=38151
k=0052 9907 7324 7081 4041 1152 8009 2093 7578 8809 9160 9982 8267

n=13, x=22, y=911, z=2341
k=0001 3516 7993 8504 3164 4546 2873 0759 3214 8779

n=13, x=23, y=9825, z=16748
k=0021 5313 6597 0930 7172 7377 6706 1852 3711 0173 9576 1346

n=13, x=26, y=7307, z=12403
k=6968 8480 3252 9579 5303 5340 6882 9158 2918 5090 8340

7.7.18 n=13, x=27 to 9999

n=13, x=27, y=7850, z=24077
k=1790 5846 1200 6039 1144 4301 1839 3047 8396 3754 0538 4971

n=13, x=29, y=6243, z=17843
k=0057 5205 0119 0478 8662 3449 5693 9918 3317 6840 0095 9191

n=13, x=44, y=711, z=2735
k=0005 5997 1185 5261 1303 2867 3923 8715 6088 0454

n=13, x=53, y=79, z=443
k=0001 3644 0013 4995 6011 1623 5358 6191

n=13, x=53, y=1027, z=6280
k=0006 9130 4544 1244 1094 9936 8926 3965 5342 4026 0107

n=13, x=53, y=1048, z=1797
k=0203 9611 8798 1311 9910 2880 7811 2991 2822

n=13, x=53, y=2028, z=2549
k=6641 7018 0057 4239 5458 1209 3070 8673 7808

n=13, x=53, y=3042, z=10349
k=0936 1501 4935 9739 4152 8697 9496 5080 9312 7992 3436

n=13, x=53, y=3797, z=5530
k=4033 6541 1596 2973 7924 9995 4942 9279 9391 4415

n=13, x=53, y=7531, z=62562
k=9007 3900 5711 4361 9709 6864 3705 1288 8084 6739 4727 3954 2288

n=13, x=71, y=9289, z=35204
k=0005 4938 5032 3802 0412 0391 1868 3871 8980 1318 2271 4471 6884

n=13, x=79, y=636, z=655
k=0003 9463 2620 0238 0800 7969 7616 6064

n=13, x=79, y=1014, z=8191
k=0113 8618 0518 2938 9974 6737 7560 6635 2985 8937 3928

n=13, x=79, y=2028, z=6943
k=0007 8319 6778 0870 9964 5825 9371 6690 6478 3438 2316

n=13, x=79, y=2544, z=6751
k=0004 4593 5200 8764 4270 1117 1674 5611 2324 4667 7603

n=13, x=79, y=3251, z=29415
k=0001 6337 3710 1824 8513 8802 2985 1562 6450 4859 7413 1716 5951

n=13, x=88, y=265, z=3433
k=0114 9098 0694 8223 4633 4976 6827 8722 7567 0676

n=13, x=91, y=157, z=1613
k=0217 1057 6906 2699 1426 8809 2773 2730 5975

n=13, x=92, y=4069, z=6553
k=0001 6715 5110 0930 4610 9367 1016 7007 3910 4329 4083

n=13, x=101, y=3611, z=13004
k=6411 7213 8612 2489 6288 8873 5086 1917 4070 2041 5328

n=13, x=106, y=521, z=6017
k=0004 0777 1200 0868 2766 7760 7990 5869 2885 0984 1980

n=13, x=106, y=5731, z=11909
k=1339 4762 8131 3494 9493 3026 7597 8418 7935 8107 6803

n=13, x=117, y=5035, z=49612
k=0377 4465 7049 4415 6687 9974 1520 9168 0346 2572 2800 1707 8584

n=13, x=121, y=795, z=2731
k=0001 7894 0174 2381 4558 9768 9522 2692 7882 8279

n=13, x=131, y=4056, z=8867
k=0044 4564 3039 5108 4032 6247 0235 8295 5101 4090 2140

n=13, x=131, y=7189, z=30161
k=6017 4024 2856 7241 5480 0494 8787 5845 8839 5685 7743 1479

n=13, x=133, y=4187, z=16024
k=0005 1463 1804 2104 9940 0039 8578 8513 5778 1380 7506 4516

n=13, x=156, y=677, z=6917
k=0011 3579 5231 7263 2047 2736 6382 6001 5037 1493 0376

n=13, x=157, y=1484, z=10349
k=0647 8091 6801 0679 2984 3494 7641 6358 9521 5628 6124

n=13, x=157, y=2173, z=18730
k=0054 6378 9381 3479 7168 4077 3125 0028 2861 7562 4447 9327

n=13, x=157, y=7436, z=20749
k=0054 5414 0435 5916 2620 6735 2156 5084 1684 7684 2539 5332

n=13, x=158, y=6943, z=32917
k=0001 4751 5860 4351 9606 9220 3765 0087 0608 4667 7569 7240 9447

n=13, x=169, y=4187, z=8867
k=0033 3814 8355 5229 1406 7884 7823 4051 2786 3854 7831

n=13, x=237, y=1093, z=3829
k=0038 3403 3047 1581 4017 1045 0164 5431 3173 6391

n=13, x=248, y=553, z=6009
k=0001 6160 3080 8459 5026 7279 9414 9728 5783 2102 3838

n=13, x=313, y=4892, z=45457
k=0050 8368 2973 6439 8550 1755 5653 5793 4420 0605 1590 1645 6386

n=13, x=359, y=2215, z=22319
k=0192 1447 8738 2031 8686 4398 6379 8014 5234 4612 7696 4447

n=13, x=390, y=6553, z=15913
k=0001 0316 1043 1468 3841 7721 2703 2824 1757 1628 3217 8488

n=13, x=463, y=785, z=8193
k=0025 1689 3334 1259 8651 8108 0045 8012 1276 0859 3311

n=13, x=507, y=677, z=911
k=0093 1434 1128 8341 2190 4314 9934 6583

n=13, x=521, y=1822, z=5609
k=1021 5014 5475 2212 4335 7161 8735 3547 5711 3552

n=13, x=521, y=9127, z=89043
k=0005 2243 4236 7707 4161 1749 8161 5986 3702 5428 2088 0705 9135 8015

n=13, x=524, y=939, z=6083
k=5216 9529 2974 0378 2433 4267 4598 4151 4483 0490

n=13, x=524, y=9899, z=39683
k=0002 9398 9179 7548 3932 1749 2790 4875 6373 1283 0475 2583 2330

n=13, x=551, y=4839, z=13655
k=1576 1321 5426 7677 5615 9935 1085 2658 4084 7778 5519

n=13, x=583, y=4792, z=16375
k=0001 3304 2467 0926 4439 6349 2155 7413 9170 0056 7672 3186

n=13, x=845, y=8012, z=22117
k=0020 2356 0354 8844 4223 3464 8362 5328 0728 0689 7788 8058

n=13, x=927, y=4759, z=52963
k=0110 4270 5375 8771 7381 4974 8233 0924 0691 3724 6571 8620 2823

n=13, x=937, y=3823, z=7420
k=7773 6442 5941 9818 4398 4738 1683 5836 4178 4386

n=13, x=1325, y=9283, z=26533
k=0098 9763 7515 2605 7109 5770 9398 3609 2663 8433 8812 2135

n=13, x=1365, y=2731, z=8191
k=0002 4467 4374 2965 6792 5400 8351 1646 5905 9718 5527

n=13, x=1435, y=1613, z=24573
k=0209 4169 8338 6999 1689 6856 9522 1159 8693 1259 4450 3767

n=13, x=1873, y=6095, z=30048
k=0474 5427 6812 0846 3657 3171 9152 7839 2152 4499 3411 1855

n=13, x=2389, y=2731, z=40955
k=0003 4130 7092 6147 1234 8337 4410 9612 2202 7359 1583 0753 3607

n=13, x=2446, y=8321, z=57831
k=0068 7537 5944 6880 4641 9734 6021 4175 6446 8632 5183 0339 1539

n=13, x=2577, y=2887, z=4372
k=0006 5188 7038 0230 9236 7720 6540 1034 5818 7446

n=13, x=3279, y=3329, z=25016
k=0055 0248 9303 5600 1451 3897 4889 3338 9931 6563 5729 8878

n=13, x=3288, y=6553, z=95329
k=0002 6141 3776 2883 6019 3198 1873 3531 9929 9763 9334 0033 0661 5623

n=13, x=3511, y=4823, z=19891
k=0002 2653 2914 6930 3453 8774 6158 6474 5859 7329 3569 9559

n=13, x=4831, y=8193, z=85489
k=3849 8415 1583 9383 9192 9043 5277 0221 5421 8856 2464 5760 1855

n=13, x=5512, y=9491, z=122747
k=0022 3619 3888 5527 8436 3732 9022 6694 9479 3173 7735 7038 2423 9641

n=13, x=7111, y=8948, z=47571
k=0002 1108 1764 8783 8117 7532 2035 8678 9284 2456 9787 0674 0319

n=13, x=7159, y=8321, z=122865
k=0019 8659 9562 2201 3784 9049 4567 3904 6047 6763 8274 8805 1608 9951

n=13, x=7289, y=8191, z=124815
k=0023 9439 8857 6154 8867 8219 1411 9538 5080 4711 9388 7457 2185 9871

n=13, x=8533, y=8971, z=137288
k=0058 5663 7410 7832 8672 2981 7692 8361 4194 5284 3498 8113 9499 3756

7.7.19 n=17, x=10000 to 19999

n=13, x=10271, y=13624, z=29511, k=46 digits : 0031 ... 2959
n=13, x=11713, y=17551, z=107498, k=53 digits : 0001 ... 5096
n=13, x=13197, y=18683, z=209666, k=56 digits : 2926 ... 7316
n=13, x=13351, y=18345, z=85906, k=51 digits : 0659 ... 6872

7.8 n=17

7.8.1 n=17, x=2 to 9999

n=17, x=2, y=103, z=921
k=0013 0102 8657 4854 2711 2945 6689 3795 3253 4778 7273 2631

n=17, x=2, y=1123, z=1195
k=0050 2256 8050 3773 2136 5487 4527 8254 6546 8667 2820 1700

n=17, x=2, y=6645, z=53567
k=3458 0575 1210 2265 3485 3104 7816 8493 7707 8552 1995 3464 5755 1789 3821 0733 3972 2031

n=17, x=3, y=3676, z=4591
k=0345 1274 0614 2492 8713 2000 6022 9607 7876 2141 8108 8862 0609 3729

n=17, x=4, y=1445, z=8589
k=1517 5877 2616 4825 6173 4386 0583 9056 1399 1508 4603 8278 3344 7122 0186

n=17, x=4, y=1535, z=10099
k=0001 9067 1412 5939 7732 3208 7597 5009 8724 6555 5147 1250 5309 5820 0180 6849

n=17, x=4, y=7061, z=21509
k=0007 4301 4531 6359 8046 6072 2267 8417 3551 9269 2628 9548 1399 5570 5207 5320 5884

n=17, x=5, y=1531, z=6576
k=0015 9751 0338 6332 5672 4182 1262 0989 5542 0477 2980 7372 0213 8174 6208

n=17, x=5, y=5316, z=42881
k=0049 1681 0820 1283 8328 2204 4163 1083 9671 7002 8371 6957 4349 5229 5564 2106 3491 7011

n=17, x=5, y=7356, z=18701
k=6084 5085 0503 0034 1295 6943 4027 7162 0197 2488 4816 0211 1994 2449 5135 3278

n=17, x=6, y=239, z=1535
k=6624 9854 0241 6505 6408 6328 8554 2657 2848 5224 5491 4264

n=17, x=6, y=2369, z=3065
k=4213 9655 9334 8281 5284 7906 1924 0757 9946 2866 2201 2432 2900

n=17, x=6, y=8365, z=43021
k=0027 4328 4325 7735 0670 8674 3004 5575 8466 6150 2188 8504 8940 2982 2970 5773 3626 6512

n=17, x=7, y=1871, z=4083
k=0045 5509 5629 3956 4171 4584 0561 9439 7538 7264 4766 5225 8830 0375

n=17, x=7, y=4083, z=6295
k=0002 1260 7148 8219 4954 3810 0182 0723 4368 3600 4568 9250 0376 6023 8351

n=17, x=8, y=4765, z=42333
k=0027 9069 1652 5687 8111 4165 4646 1946 1360 2820 8710 6510 7100 6762 5238 9110 3755 8040

n=17, x=9, y=7352, z=62441
k=8069 9266 5842 4428 8265 0764 9358 5716 0337 0792 4901 0164 5367 4470 5715 1145 3765 8980

n=17, x=12, y=1123, z=1195
k=0008 3709 4675 0628 8689 4247 9087 9709 1091 0066 8545 6294

n=17, x=13, y=443, z=3914
k=0052 6727 5843 1942 3517 2065 6529 9458 8150 1848 5927 9200 8145 9028

n=17, x=14, y=1871, z=6295
k=0002 3212 9281 6575 6093 6671 5462 1706 1125 3081 1741 5445 3846 1567 9836

n=17, x=17, y=685, z=4017
k=0039 4720 6204 8162 1969 9945 9972 7389 2539 9634 6898 9066 6822 0575

n=17, x=17, y=1838, z=16271
k=0772 3843 9849 6431 0662 1926 0706 3268 3711 3121 6857 7476 6124 6862 7823 9591

n=17, x=19, y=548, z=4291
k=0126 8826 0108 8614 7316 1308 1894 1594 5433 7433 5414 7779 4271 4092

n=17, x=19, y=927, z=2143
k=0011 2336 5596 3149 4529 8465 6599 2084 2894 5944 3310 8340 5823

n=17, x=19, y=1871, z=2042
k=0001 9895 1595 5606 5829 3689 1198 3899 1176 3307 2471 9477 9159

n=17, x=20, y=239, z=1259
k=0083 3650 0587 4988 5872 5757 7033 5127 4954 5580 0203 4567

n=17, x=23, y=1021, z=1228
k=0010 8935 6200 7669 0098 7856 9010 6931 3064 0505 1707 8731

n=17, x=24, y=9283, z=18163
k=0629 6466 8790 4095 3864 4395 4109 6840 0583 3979 7131 3360 9595 6493 7531 2866

n=17, x=25, y=7969, z=26194
k=0024 6538 8322 8501 8771 4890 3262 7358 8637 3243 1352 5497 8367 9404 8548 3451 6803

n=17, x=29, y=307, z=2163
k=0025 7850 4761 0967 3572 7094 3389 0362 2941 8078 1557 1803 0503

n=17, x=31, y=1535, z=11331

k=0001 5517 5001 8044 1419 7518 5388 5156 1485 2616 9121 2364 0896 2233 6116 2135

n=17, x=35, y=4492, z=40087
k=0002 8284 5390 3120 7187 3707 9175 5598 3477 8522 7829 9338 1113 2764 3753 4835 2739 6451

n=17, x=37, y=5726, z=48757
k=0048 1435 9809 5561 6465 5964 5512 2469 8052 3969 5170 5483 0576 6510 6413 1481 7118 3896

n=17, x=44, y=4291, z=12651
k=0002 2802 7116 9017 8981 7872 1155 2323 0992 1209 1620 5705 6970 9252 2121 0014

n=17, x=51, y=137, z=953
k=6625 1492 5828 4083 9615 6830 3251 0746 3965 7093 0895

n=17, x=57, y=1636, z=3061
k=0637 0079 1445 7068 9667 4943 4864 2492 9756 7179 5404 0055 2819

n=17, x=61, y=443, z=1236
k=0010 9789 7912 3379 6490 6146 4884 9673 2556 3132 2216 2694

n=17, x=65, y=1871, z=2456
k=0014 2688 2234 1498 2300 8412 2762 8450 3167 5725 3817 3849 2678

n=17, x=73, y=6447, z=11119
k=1159 6583 5248 4535 4087 6256 6703 2646 5648 9211 6794 8253 7116 7946 4991

n=17, x=93, y=886, z=7117
k=0005 2581 7193 4446 1792 2371 1645 7044 7542 6533 9220 6079 1360 8278 2568

n=17, x=102, y=103, z=1021
k=0001 3273 1645 0303 3692 4311 7765 4678 6671 8479 5008 0731

n=17, x=103, y=125, z=1228
k=0020 7695 2819 7267 1334 1990 9483 6636 8440 7001 1511 3607

n=17, x=103, y=255, z=2143
k=0007 5331 6242 5520 5578 5973 1481 5015 9780 5718 7868 0863 1999

n=17, x=103, y=1377, z=11119
k=3848 4027 7138 7883 6862 9754 4669 9284 1498 3543 9406 8055 9866 5280 1711

n=17, x=103, y=2312, z=21183
k=6901 9912 5951 0897 0265 6262 9656 1561 2705 0173 3511 6062 2881 3726 4647 7011

n=17, x=103, y=7352, z=65391
k=1475 8592 4313 8479 3287 8871 4263 2523 4043 7206 5461 5990 9147 8909 6050 9328 4223 8751

n=17, x=119, y=409, z=1123
k=0001 3146 3689 2366 4706 7604 1280 6560 2950 0864 8582 5615

n=17, x=127, y=5613, z=32029
k=0172 0660 9721 7781 1720 4070 4063 5946 2146 0516 6950 5499 8387 0090 0808 5237 0151

n=17, x=131, y=409, z=1195
k=0003 2276 9043 7781 5857 7855 6023 1680 2085 0414 8955 8759

n=17, x=137, y=2163, z=5975
k=0890 4908 3422 0556 0006 5408 1448 9422 4211 9716 8027 6919 5633 3239

n=17, x=137, y=2654, z=2687
k=0003 8478 4813 8259 9587 5356 8003 1847 7340 1286 0052 8554 6291

n=17, x=137, y=3673, z=14082
k=0004 7521 2596 6709 8999 3918 5137 8045 4567 7918 3328 1907 0264 1971 4400 9391

n=17, x=137, y=5497, z=6738
k=2321 6054 0157 2432 6106 1801 0231 2420 0956 4579 2182 0594 8220 3047

n=17, x=185, y=206, z=1871
k=5917 5461 1757 3189 9431 1364 5118 6538 5143 0105 2528 9803

n=17, x=229, y=1871, z=20420
k=2132 9591 3116 4660 7439 6687 1908 3711 2117 1762 4722 8465 3401 5834 8228 7385

n=17, x=239, y=301, z=2045
k=0001 3005 5463 4280 2930 3974 6694 2713 4214 5820 6782 0640 3559

n=17, x=239, y=896, z=5615
k=0455 9216 9540 4182 5060 6919 2959 3378 7681 1807 3683 2428 6946 2438

n=17, x=239, y=959, z=6427
k=3697 5690 2871 8291 1302 7964 1692 3367 3308 8258 7940 0303 8630 9287

n=17, x=239, y=1973, z=10003
k=0213 0881 5773 0557 6385 0574 5856 6433 2784 0285 2049 8440 0443 6629 0191

n=17, x=239, y=5100, z=30839

k=0054 9081 1245 3236 7417 1956 3854 8745 9568 7058 3857 0210 7511 8084 1375 1694 8182

n=17, x=239, y=8061, z=25481
k=0001 6393 6878 7573 9588 1788 8821 1311 0364 3854 2426 3056 3994 7403 6761 7125 0039

n=17, x=279, y=2143, z=19999
k=1095 2319 6898 8190 6626 4576 6831 1531 9816 7071 7070 9124 4116 8705 5451 6799

n=17, x=289, y=9183, z=22681
k=1848 0454 9577 0439 6795 6658 0186 5150 2788 8945 3730 7858 3119 0376 8225 9647

n=17, x=307, y=953, z=2452
k=0005 8357 4264 9209 4596 0398 3436 9765 0919 1949 9982 8875 8531

n=17, x=307, y=1629, z=14968
k=0012 6929 0215 0354 7595 4452 2475 8686 0737 4653 7161 1622 7315 7164 9271 9038

n=17, x=307, y=7635, z=69022
k=1132 0176 8859 5698 0243 6111 3340 9690 4283 0678 9680 8080 7258 9662 6631 7710 4724 3227

n=17, x=409, y=3911, z=7962
k=0001 6305 7656 8707 8746 2352 8114 9288 9100 4796 5109 3260 7635 9893 1504

n=17, x=647, y=6358, z=14111
k=6007 5763 7746 9870 7652 4126 9829 5064 0944 7668 1561 3340 7422 0327 8276

n=17, x=721, y=3684, z=31285
k=0031 7047 0107 0447 5716 4206 8905 4425 8374 4559 9133 6840 9023 9104 5725 8187 0781

n=17, x=819, y=8083, z=57223
k=0019 9651 9153 5294 6037 5468 2537 6041 1340 3564 6655 2990 7116 7088 7498 2983 5998 8671

n=17, x=824, y=3743, z=34511
k=0131 2724 7108 0815 7119 7744 4233 2905 6198 8025 7715 4250 4586 6408 1988 2592 1802

n=17, x=886, y=9195, z=81241
k=4419 9638 0434 0583 6404 1105 8793 4559 5515 1936 5566 8181 3319 6361 9815 3483 4852 2263

n=17, x=953, y=8652, z=39593
k=0442 2676 2297 7438 5375 2313 2461 1931 4356 3762 2102 9158 6079 8539 7038 8998 4276

n=17, x=1021, y=3400, z=31621
k=0028 7809 3799 0661 7744 9740 0654 9395 7943 1072 6949 5045 2864 1829 4007 4989 8793

n=17, x=1122, y=2143, z=11119
k=0227 0056 4432 2365 2320 3703 9564 5959 8387 2863 9502 6993 5074 3389 2616

n=17, x=1339, y=6326, z=44065
k=2385 5615 5243 9136 7366 0689 6516 1689 9255 4782 1308 9895 9510 7355 5404 9597 1787

n=17, x=1347, y=4765, z=26317
k=8248 0557 3225 6583 8189 1946 9639 1422 7159 7531 8173 9240 2832 7050 2777 2359

n=17, x=1644, y=6805, z=59269
k=0020 7258 1674 3645 1579 5084 4272 5498 7648 6698 2160 7395 7747 6022 7418 8581 9675 3300

n=17, x=1871, y=6819, z=63242
k=0051 3209 9355 9232 2587 2462 4152 2883 1595 4967 9464 9147 4736 9249 0866 8022 2815 5559

n=17, x=2045, y=4799, z=30839
k=0006 8196 2555 0479 1388 4056 5440 0727 8252 3308 1684 3626 4192 8473 0646 2432 9759

n=17, x=2215, y=7356, z=77131
k=0963 0480 1626 9528 4291 5134 5589 3952 3849 2655 9135 7409 8005 8451 4485 2166 1729 5795

n=17, x=2329, y=3469, z=47737
k=9001 3608 6448 5407 5733 5871 4020 7078 3070 5928 9462 6863 2925 0162 9507 9720 8687

n=17, x=2552, y=2859, z=43691
k=2416 4451 0685 6481 8423 8354 7162 9570 5916 6038 9693 7257 1970 5529 6307 1129 1280

n=17, x=2859, y=9517, z=29767
k=0001 3965 1901 3820 0742 0921 3704 2676 9930 6503 8125 1498 5014 7356 6070 4114 2231

n=17, x=2927, y=8168, z=34511

k=0016 9348 8484 3016 3586 4885 4040 3300 9349 2422 3061 6050 2969 1302 9551 9471 5036

n=17, x=3003, y=7484, z=34511
k=0018 0148 8890 6943 1347 9455 7383 3437 3879 4566 8867 9635 2953 0125 7361 4602 6287

n=17, x=3299, y=9527, z=28919
k=7613 6666 2519 2533 3667 5735 4039 9651 7547 7829 4057 2942 8912 3895 6451 3039

n=17, x=4204, y=5615, z=30839
k=0002 8352 5499 0550 3143 0685 5415 9026 9792 8364 5841 4204 0825 1363 1630 8554 3638

7.8.2 n=17, x=10000 to 19999

n=17, x=11119, y=14994, z=131071, k=74 digits : 0045 ... 7048
n=17, x=14111, y=17154, z=40391, k=66 digits : 0020 ... 5444

7.9 n=19

7.9.1 n=19, x=2

There are no solutions (2,y,z) from y=3 to 4000 for x=2, n=19.

7.9.2 n=19, x=3

There are no solutions (3,y,z) from y=4 to 5000 for x=3, n=19.

7.9.3 n=19, x=4

n	x	y	z	k
19	4	177	1597 (x)	55 digits : 644 ... 7801
19	4	4113	14881	71 digits : 778 ... 4700
19	4	4114 to 9999	none	

(x) denotes that the solution is an ‘extended form’ of the ‘x plus y’ solution, see Section (5.3.1).

7.9.4 n=19, x=5

Prime x,n

n	x	y	z	k
19	5	31	191 (x)	39 digits

19	5	382	1597	55 digits
19	5	6852	22877	74 digits
19	5	383 to 9999	none	

7.9.5 n=19, x=6

n	x	y	z	k
19	6	59	419 (x)	45 digits
19	6	229	955	51 digits
19	6	7867	9463	67 digits
19	6	230 to 2000	none	

7.9.6 n=19, x=7

There are no solutions (7,y,z) from y=8 to 4000 for x=7, n=19.

7.9.7 n=19, x=8

n	x	y	z	k
19	8	687	2855	59 digits : 288 ... 2781
19	8	3307	23307	75 digits : 155 ... 9484
19	8	3308 to 5000	none	

7.9.8 n=19, x=9

n	x	y	z	k
19	9	647	1142	52 digits : 1874 ... 3416
19	9	3308 to 5000	none	

7.9.9 n=19, x=10 to 9999

n=19, x=11, y=916, z=1103
k=0562 5352 0218 1527 2575 7392 5484 6238 6340 5996 7805 9040 8055

n=19, x=13, y=5496, z=29533
k=4088 3314 7835 1568 6192 2447 9443 7607 6696 3265 3765 1604 3836 6050 6858 1800 3056 7672 2946

n=19, x=13, y=6159, z=7069
k=0002 2495 9190 9149 7990 2944 3408 5140 8422 2738 0649 0845 3761 4528 3515 1454 9031

n=19, x=19, y=7829, z=9463
k=0242 1287 2163 2539 6802 5131 8434 0833 7097 9387 9324 5168 9431 7886 7143 9255 1783

n=19, x=21, y=2357, z=11933
k=0004 8631 1628 0473 5337 1184 1958 0365 9866 6993 7804 1894 8548 8115 2902 8872 2366 7423

n=19, x=23, y=457, z=3079
k=0588 4781 7967 8309 1293 2793 5980 2031 9927 2944 0974 2145 3784 6635 8431

n=19, x=23, y=3805, z=36948
k=0018 8171 7101 8895 3051 0039 3441 5232 0463 8141 1851 0483 7571 5950 3513 8127 4626 7128 5258 0315

n=19, x=24, y=4351, z=11215
k=7543 7392 8986 2049 5767 2971 8425 9388 2837 1588 0459 8615 2157 8166 2608 8608 8250

n=19, x=24, y=7067, z=13307
k=0010 0923 8678 5558 6958 3962 1473 5507 4239 3586 8810 6340 0386 5323 9466 8054 3668 7436

n=19, x=39, y=761, z=6689
k=0002 4213 2041 2867 2919 1295 9729 3490 1203 8146 7717 6123 8060 3166 5540 3257 1439

n=19, x=41, y=3664, z=10265
k=1065 9132 0784 5748 6401 2780 3649 5943 0834 8036 4314 7317 7171 2419 5170 3275 7993

n=19, x=43, y=457, z=1145
k=0582 2561 6160 8282 0559 1939 0068 7977 1304 0660 9360 3921 7975

n=19, x=45, y=2357, z=22877
k=0027 7722 5875 6132 4621 7576 8723 2261 0524 5788 2952 4940 6609 8566 7192 2102 2863 0209 8159

n=19, x=53, y=1832, z=13757
k=0032 0793 7749 1019 0917 7399 6908 7596 4278 1333 4756 1708 2658 9476 1915 8979 4020 1639

n=19, x=76, y=2699, z=10831
k=2051 2449 6564 8507 1039 9609 2302 1482 6847 3755 8611 1605 7557 7847 2097 4998 2209

n=19, x=149, y=1217, z=12541
k=0003 2471 4868 0884 7148 8149 6613 8396 1095 6451 7146 9099 4238 1347 4997 4210 2338 2903

n=19, x=183, y=7985, z=22808
k=0001 9091 6588 9140 2502 6319 1046 0091 8628 6030 4934 3193 2633 1493 4398 7944 8254 5013 4925

n=19, x=191, y=1216, z=6271
k=0968 4907 3799 9677 5246 4384 8084 1549 7336 8680 6996 0880 5543 4678 4747 3759

n=19, x=205, y=1146, z=6271
k=0957 4672 6618 5065 8983 2822 4491 1262 6046 9108 8700 6007 7161 2450 2897 7475

n=19, x=229, y=764, z=1713
k=0009 2194 1995 4782 8908 9997 0840 3960 9014 4419 7599 0530 5199 9783

n=19, x=393, y=2851, z=6388
k=0280 0131 7826 7802 0164 4578 2796 9912 5313 7419 4247 5036 0279 5616 4956 5141

n=19, x=509, y=6088, z=18813
k=0281 2241 2396 9789 8633 9083 1344 1161 4343 7456 9416 8590 8827 3032 4293 2594 2565 2816

n=19, x=571, y=2101, z=9349
k=0024 8147 9635 1174 7115 2052 0104 7617 8460 1833 0690 1304 4456 4581 3000 5927 4623

n=19, x=601, y=4584, z=19609
k=0666 9280 0827 2142 0286 7464 8900 9438 2634 7370 2334 9498 7676 1556 7041 3963 5765 3304

n=19, x=916, y=4113, z=20437
k=1026 7256 8255 3343 2676 7096 1854 1214 9332 4920 8488 6322 8570 9069 8407 3570 7451 8935

n=19, x=1559, y=2281, z=18336
k=0154 3516 9560 1937 0241 3765 2643 4633 8635 9051 2933 4107 4208 7001 8964 1933 4919 9624

n=19, x=4529, y=5703, z=14327
k=2504 3152 1873 4296 1181 8296 7828 7122 9629 7159 0065 8685 9160 3582 9593 9717 6623

n=19, x=5932, y=6085, z=27877
k=0002 8639 5972 6330 5376 7479 8144 3450 2273 9134 0019 8638 7968 8793 8130 4757 5581 2420 6088

7.9.10 n=19, x=10000 to 19999

n=19, x=10057, y=11404, z=101917, k=83 digits : 0122 ... 7141
n=19, x=10352, y=18893, z=115597, k=83 digits : 0694 ... 4364
n=19, x=10913, y=17032, z=39537, k=75 digits : 0299 ... 8734

7.10 n=23

7.10.1 n=23, x=2

Prime x,n

n	x	y	z	k
23	2	5	47 (x)	36 digits 6111 5711 8472 4799 8030 7644 8720 2689 0707
23	2	15	47 (x)	36 digits : 2037 1903 9490 0307 2099 1633 4245 4607 2944
23	2	829	967	47 digits
23	2	973	9165	84 digits
23	2	1321	9249 (x)	84 digits
23	2	2347	12267	Note *
23	2	4835	32477	Note *
23	2	9293	52039	Note *
23	2	9294 to 9999	none	

(x) denotes that the solution is an ‘extended form’ of the ‘x plus y’ solution, see Section (5.3.1).

Note *. The full k-values are given below for these solutions.

n=23, x=2, y=2347, z=12267
k=190 8538 6124 3449 9809 2464 7858 4991 9854 8435 9774 6950 6660 0222 8339 6033 1884 9579 9328 8960 6302 1984

n=23, x=2, y=4835, z=32477
k=1858 8461 3487 2606 3757 4648 5682 4842 4616 7526 4434 5823 8894 1806 4854 1288 0930 5911 2925 3183 9365 2992 6092 7155

n=23, x=2, y=9293, z=52039
k=3090 6569 3137 0295 2157 2699 3844 9195 1699 8582 4284 3991 6596 1078 9144 0341 8740 7275 4451 7105 4121 6611 0422 8435 5851

7.10.2 n=23, x=3

Prime x,n

n	x	y	z	k
23	3	11	47 (x)	36 digits : 1851 9912 6809 8418 4095 0587 8500 6453 0415
23	3	13	94 (x)	42 digits
23	3	440	3083 (x)	73 digits

23	3	829	1504	67 digits
23	3	2350	4003	Note *
23	3	2393	6533	Note *
23	3	3227	20366	Note *
23	3	1322 to 9999	none	

Note *. The full k-values are given below for these solutions.

n=23, x=3, y=2350, z=4003
k=2536 8324 7204 1536 0816 5471 3891 6592 2993 3710 7794 2293 3991 4212 5673 5333 7626 0371 2720

n=23, x=3, y=2393, z=6533
k=0001 1923 7957 1360 1460 9485 6768 1338 8103 7776 4793 7741 8270 4737 0873 1664 1510 3498 3502 3415 3679

n=23, x=3, y=3227, z=20366
k=0645 6632 1956 5137 7117 6285 5068 1928 4223 3789 2142 4328 0784 8361 9045 3750 0714 0735 7193 4746 5493 0506 0391

7.10.3 n=23, x=4

n=23, x=4, y=5981, z=27069
k=0013 6632 7697 3738 6173 2711 4242 8695 9012 7605 6806 2657 1231 5345 3492 9430 5512 2311 2664 1073 9886 9372 4612 4584

n=23, x=4, y=8507, z=38919
k=0002 8295 6957 8645 5194 3272 6182 9587 4823 3664 5911 8811 1353 6657 1350 5475 2422 5736 0975 5016 2622 0757 8206 0481 4461

7.10.4 n=23, x=5

Prime x,n

n	x	y	z	k
23	5	7	47 (x)	36 digits
23	5	536	3221 (x)	74 digits
23	5	1088	6533 (x)	81 digits
23	5	1363	1658	67 digits
23	5	1934	13019	87 digits
23	5	1939	7614	82 digits
23	5	1940 to 9999	none	

(x) denotes that the solution is an ‘extended form’ of the ‘x plus y’ solution, see Section (5.3.1).

7.10.5 n=23, x=6

n	x	y	z	k
23	6	139	1363	67 digits : 109 ... 0436
23	6	1013	2531	72 digits : 1226 ... 4771
23	6	1457	7601	82 digits : 27 ... 3776
23	6	1458 to 9999	none	

7.10.6 n=23, x=7

n	x	y	z	k
23	7	278	2679	73 digits : 1 ... 9016
23	7	461	1833	69 digits : 1 ... 8143
23	7	1385	5452	79 digits : 165 ... 1436
23	7	5153	6063	Note *
23	7	1386 to 9999	none	

Note *. The full k-value is given below for this solutions.

n=23, x=7, y=5153, z=6063
k=0448 2617 2911 1802 4505 1425 3374 5888 6231 7953 4872 5508 5548 3425 4719 3336 2985 6471 4391 2799

7.10.7 n=23, x=8

n	x	y	z	k
23	8	139	987	63 digits : 674 ... 7638
23	8	277	829	61 digits : 7 ... 1971
23	8	1386 to 9999	none	

7.10.8 n=23, x=9

n	x	y	z	k
23	9	599	1112	64 digits : 1917 ... 8300
23	9	600 to 9999	none	

7.10.9 n=23, x=10 to 46

n=23, x=10, y=277, z=967
k=0172 5451 1200 4507 1793 5846 0923 7783 3937 6876 0163 4703 6175 1340 8196 7987

n=23, x=11, y=5969, z=38111
k=9243 0076 7300 2698 9979 0015 3383 9843 3119 7373 7898 7495 8264 3955 7829 1385 7314 6240 1417 4571 6708 8727 1289 4159

n=23, x=12, y=235, z=1807
k=0001 5952 5549 0437 9119 3175 7467 9553 5956 5882 5721 6437 3943 4651 0437 4911 9043 8781

n=23, x=12, y=6533, z=39185
k=0001 4267 3337 3068 2840 1192 9261 9542 0627 9244 1772 5825 0386 4024 7466 6660 3443 3299 3673 1297 8582 6009 9925 4736 2111

n=23, x=13, y=141, z=973
k=0298 7583 2847 8566 0112 6173 0079 0856 1587 2027 1783 9380 0462 5539 3198 5711

n=23, x=16, y=139, z=235
k=0006 5517 2586 9249 4668 1757 6948 0319 8569 9350 8519 0489 8154

n=23, x=16, y=4277, z=17397
k=0002 8521 7890 4399 2338 1476 4601 4553 9397 2257 8552 4596 4179 1809 2133 1208 9830 1395 6533 2784 3327 5811 8636

n=23, x=17, y=1385, z=1657
k=0279 3270 4252 9036 8154 0573 8939 9386 5312 8690 2406 2959 2732 8633 8150 5442 2447

n=23, x=17, y=6078, z=24863
k=4874 9261 6411 6013 2204 5150 7579 1264 6432 2639 3162 9575 0706 8218 1091 4971 9804 1968 8860 4414 4940 0014 8039

n=23, x=21, y=139, z=1504
k=0271 7805 0585 0963 9665 5974 4854 3455 7078 2867 9817 3480 7492 4116 2523 2474 2509

n=23, x=23, y=2397, z=8285
k=0028 9094 2212 4327 3633 4992 4193 2945 4811 7485 6025 6220 8981 1824 2774 9829 3433 6610 1729 2867 0551

n=23, x=24, y=2531, z=7091
k=8551 8739 5720 2299 1403 2972 3147 6781 9727 6919 6830 2706 8447 9219 4059 9961 9659 9827 7220 1714

n=23, x=28, y=3197, z=6445
k=0709 5835 9504 1951 1651 9289 3363 8745 9006 9928 7291 0483 0355 8290 7742 3674 3637 4509 0347 6140

n=23, x=30, y=9667, z=21667
k=0084 1705 6764 8176 0129 9073 7466 2964 5601 3028 1303 5681 0302 4813 8247 1135 1753 3286 6320 5568 3075 4352 9200

n=23, x=35, y=277, z=1222
k=8492 1707 2447 0148 5299 9583 6361 4183 7826 6596 6928 5683 9217 6690 4959 7996

n=23, x=36, y=461, z=1289
k=0001 6053 1681 8067 4717 1775 9910 7226 0651 7896 3512 7925 6386 0736 3839 8097 7353

n=23, x=41, y=1128, z=8057
k=0018 6518 1262 0454 8843 8302 2188 8793 5648 7354 0582 1660 5215 4788 0366 8324 4096 8593 9962 3739 9070

n=23, x=45, y=1807, z=10627
k=4686 6692 6128 3588 6716 2328 3554 4190 7051 2207 0618 2633 6348 5699 1508 9598 1904 5317 4835 2745 4783

n=23, x=45, y=2531, z=18776
k=0009 1783 1642 5400 5485 9036 6023 2496 5007 0736 9148 0270 5555 1894 3909 1192 0063 0772 3435 0024 3658 9672 7978

n=23, x=46, y=277, z=1013
k=0104 2725 9868 5842 7351 3151 9055 3476 7411 4353 8049 2768 8946 9793 0289 3168

7.10.10n=23, x=47

Prime x, n, $2mn+1$ form, x=2.11+1, n=11, m=1

n	x	y	z	k
23	47	49	831	61 digits
23	47	108	695	59 digits
23	47	529	3221	73 digits
23	47	691	834	60 digits
23	47	1108	10703	84 digits
23	47	1383	9035	83 digits
23	46	Note *		

23	47	checked to 9999	none	

Note *. More solutions follow:

n=23, x=47, y=1668, z=3227
k=0001 9919 7197 2446 0406 1680 0813 0963 4920 1852 6345 3605 4622 6421 3985 2366 1722 6652 1509

n=23, x=47, y=2073, z=8741
k=0053 1676 5246 7918 9225 4761 3429 5001 4252 7943 0208 9977 5175 1641 7880 6821 5364 0192 7314 8794 4311

n=23, x=47, y=3868, z=4031
k=0070 3020 8437 9542 4003 0562 7701 4095 2982 6588 9746 6249 9810 4767 8553 5559 4435 1516 8036

n=23, x=47, y=4232, z=21023
k=0063 1887 3625 8335 3053 4479 3683 4890 5786 1948 1977 2786 9228 3425 9305 2231 3690 4459 5325 7895 8949 7680 3378

n=23, x=47, y=4837, z=8741
k=0022 7861 0878 1188 9870 1015 6602 5290 4540 2535 9917 4380 5167 8246 2096 4215 7581 9158 2987 5694 0855

n=23, x=47, y=9522, z=72911
k=0021 4102 2197 8833 3670 1122 8217 3295 5022 1488 9002 9143 3231 0586 8256 7995 8831 4835 7883 4814 0170 3312 8780 2419 8751 8640

n=23, x=47, y=9948, z=31055
k=0014 3568 8111 9069 4253 4106 0013 3737 9387 8305 0503 9016 5661 2037 5965 8699 7185 9462 6399 4575 1330 8046 2787 4632

7.10.11n=23, x=48 to 9999

n=23, x=67, y=973, z=4792
k=0001 4360 9167 1056 8834 2891 4869 7513 1236 6765 4380 0499 9505 0234 2114 6277 2657 6090 6787 4914

n=23, x=92, y=3221, z=6533
k=0288 8683 0327 2664 9026 3503 6911 8733 1599 8556 9321 0647 4001 7076 1987 4085 2269 7192 8341 5548

n=23, x=94, y=427, z=3083
k=0001 4251 5406 3231 9444 5663 0746 9911 5253 8929 6651 3553 1764 3868 5262 9670 2447 3437 0480

n=23, x=94, y=9663, z=12661
k=0001 9732 7375 4458 0820 5716 1991 7602 3145 8860 3568 5591 9331 7625 2486 3389 4026 6957 8837 3624 0962 9075

n=23, x=103, y=6533, z=30223
k=0005 4887 9085 0125 0616 3129 7364 6138 0180 1932 8822 4849 7374 8354 4069 8298 1703 4804 7563 0396 4878 8054 6316 3639

n=23, x=105, y=3911, z=7376
k=3010 1120 4938 9654 1357 5257 8529 3767 7369 8225 9726 4431 9982 7204 5074 1911 7240 9359 0955 0209

n=23, x=120, y=4877, z=36557
k=0414 9713 0897 5403 8606 8773 2534 1053 1563 9053 8611 2429 0707 1306 5557 1198 2023 8044 8535 2997 2098 5174 0763 1672

n=23, x=124, y=611, z=2347
k=0018 6992 8822 5913 1656 0789 0876 9239 2033 2840 7027 5109 3366 9362 8404 4925 1079 4946

n=23, x=131, y=4835, z=30511
k=0007 1841 2611 8638 5338 7777 1614 8003 7114 7684 1251 1854 0725 0450 6597 6242 6994 7713 8244 9919 9328 4161 2699 6319

n=23, x=139, y=967, z=7426
k=0001 0670 0760 1108 1249 7568 4769 4326 3056 1359 5701 7946 5268 3206 6862 8435 1884 3455 5379 8613 0863

n=23, x=139, y=2661, z=16105
k=0966 2207 6493 1735 2650 0048 5640 7014 9329 6577 0534 2810 6793 6907 9445 6417 5659 1586 1931 3086 5525 1655

n=23, x=139, y=3174, z=5521
k=0004 7837 3849 8760 5509 5731 0142 2703 1603 0399 0367 5815 8863 8617 5690 1147 8080 0518 4012 8503

n=23, x=139, y=5405, z=16584
k=0906 4708 8879 8716 7571 4429 4493 8378 7057 0074 5193 9173 4380 8076 6561 7611 9354 3765 9718 7182 1858 5787

n=23, x=139, y=7736, z=52875
k=0075 8537 5892 9542 3556 8044 7239 1263 0256 0169 1739 2153 6360 8270 8231 9960 4410 4105 4182 3232 0492 6553 6352 9017 3888

n=23, x=139, y=8310, z=42529
k=5867 5654 1883 1140 9893 3939 7573 2084 8975 0145 7374 7794 6075 2417 8661 9530 2339 5334 8727 3181 6575 6771 1729 1287

n=23, x=188, y=691, z=6947
k=2546 2713 2125 9474 5988 5887 8483 7274 4693 7589 5602 8578 6371 4233 4888 0904 3790 2555 3278 4380

n=23, x=188, y=1383, z=9431
k=0105 9962 1500 2091 9518 1460 1924 0626 9452 2027 7666 9770 3504 1341 5122 5859 0785 8417 6230 6113 9068

n=23, x=213, y=1657, z=5065
k=8975 2340 2573 2563 8346 2871 6696 7425 2570 1346 6844 3816 0108 3144 4777 5838 8414 8875 9879

n=23, x=235, y=296, z=4971
k=0003 0158 0471 4053 5849 9193 0076 4231 5039 6443 1426 2898 9935 2907 7169 2938 7351 9870 4505 3040

n=23, x=277, y=423, z=2995
k=2581 7254 2368 9649 4738 3061 4631 9756 8053 1901 6302 3003 8239 5651 9247 9269 4355 2655

n=23, x=277, y=1587, z=9661
k=0106 5194 9788 7428 6878 7844 5293 7810 8104 8753 2456 6869 7502 8007 3773 9241 0549 7928 1751 0956 8655

n=23, x=277, y=1739, z=11599
k=5425 9116 0747 4701 1910 1702 0536 6919 3566 2861 5191 5541 4877 0102 4567 0935 4767 4912 1384 5362 7351

n=23, x=277, y=3227, z=13724
k=0011 8385 0527 4567 0576 5527 7378 3182 1441 7898 5871 5712 3028 2923 7904 1922 8556 2647 2842 4331 6182 4748

n=23, x=277, y=5828, z=56573
k=0223 5647 3354 8587 0726 2098 6635 9310 7981 1089 8437 2766 6399 6522 6523 8106 5260 4247 9998 4693 3591 7326 8827 8054 7414

n=23, x=278, y=831, z=5981
k=0053 1346 5112 0031 7885 9752 1500 9295 8770 9970 2601 1546 8600 8039 0780 3637 5718 1437 9338 1131

n=23, x=331, y=5993, z=10627
k=0192 1152 6616 5325 7463 9795 8974 5778 9045 0880 6787 8113 8588 8342 4359 1093 6903 3124 6610 1080 6135

n=23, x=421, y=695, z=3221
k=5123 0553 2103 9250 0467 8603 2422 6706 0754 4068 4604 9003 9486 3026 3826 2943 4270 8295

n=23, x=461, y=1939, z=9400
k=0028 6770 2168 4563 5285 4960 5591 3131 5910 0201 8334 2781 0301 7879 5743 7707 0716 5070 9566 6051 5308

n=23, x=461, y=4232, z=6533
k=0043 8744 7254 9095 5138 2271 8551 7543 4618 5470 4394 0248 5490 5770 2619 0883 0590 9631 0674 4993

n=23, x=503, y=4971, z=13019
k=0001 3264 6576 5083 6081 9458 3574 7043 7896 6797 7279 3029 2984 1026 6584 3888 5978 5613 3193 3299 2239 3543

n=23, x=517, y=4155, z=18862
k=5381 0709 7315 1552 2249 0744 6505 4816 3725 0225 1916 0901 2518 9401 6641 1269 8843 5387 6164 3655 5517 3152

n=23, x=599, y=1381, z=2068
k=1057 9156 0173 2349 8449 9859 5625 3974 4683 8232 1296 4343 1723 4680 1019 5539 3901

n=23, x=752, y=3039, z=19919
k=0001 6785 6110 1659 9928 0708 9446 1633 6907 6259 7581 3962 1481 3798 1863 9503 1261 2819 3141 2734 6970 0918 6151

n=23, x=967, y=1112, z=13959
k=0014 2976 1170 2653 0972 4223 8802 8089 6827 7218 8496 9552 6482 0783 6562 6507 1805 2519 0585 5591 9423 4808

n=23, x=973, y=6166, z=13019
k=5528 2976 5697 5592 3800 2766 6277 2024 4458 2306 6141 0345 7917 5364 7837 3306 0179 1196 2101 5294 6763

n=23, x=1108, y=9035, z=64743
k=0701 0905 4603 5744 4945 6594 3760 6736 9058 2606 2334 0881 3261 4848 5726 8714 8278 6162 2141 8161 1141 6480 9281 0294 4453

n=23, x=1151, y=5065, z=6956
k=0058 3393 2574 4051 9869 8676 2071 8200 1484 1879 3546 8661 0787 2074 2002 5919 0102 5938 4984 2106

n=23, x=1374, y=8285, z=30269
k=3354 8906 4425 8929 9901 3461 7471 6184 3841 1786 3110 7263 2773 7168 7637 5739 3198 0050 1829 2916 9571 4071 6656

n=23, x=1380, y=2347, z=21667
k=0007 5366 9702 7793 7629 9621 3818 5730 7936 0209 9733 1902 7320 3536 5804 6848 6686 2216 2149 4131 6215 0373 4722

n=23, x=2116, y=6073, z=7177
k=0051 5709 7913 9491 7853 2688 9228 0985 4537 5093 7628 2835 2574 5503 9569 4288 6610 2292 7991 5020

n=23, x=2224, y=4183, z=14951
k=0007 4838 0848 6754 7457 6522 9889 4144 8368 8709 5146 0671 3745 7854 7763 0129 6169 4294 8080 2910 7364 3845

n=23, x=2665, y=4971, z=47611
k=6129 4244 4463 5048 1968 9092 4586 9415 5079 3384 0695 4267 7035 0437 9961 7158 6291 3854 1418 2683 4944 9379 7635 2743

n=23, x=2716, y=4155, z=47611
k=7195 4817 9585 2398 2783 9101 2643 0992 5069 7217 1165 4645 4293 7633 2411 2233 4577 4964 6797 8998 5578 9540 2277 6432

n=23, x=3384, y=5755, z=35779
k=0007 7686 3428 1712 1318 4413 6593 7288 2872 6988 1287 6232 5851 2196 1755 1102 1344 2251 4420 9101 4589 2855 9616 5017

n=23, x=8883, y=9293, z=26681
k=0028 8224 3465 7622 6625 8376 9696 0412 7135 6189 4020 3300 7873 9174 3557 8630 0172 2342 7200 7518 1586 3044 3223

7.10.12n=23, x=10000 to 19999

n=23, x=10359, y=16012, z=47611, k=95 digits : 0489 ... 7573
n=23, x=12156, y=13019, z=28775, k=90 digits : 0079 ... 2611
n=23, x=13019, y=18901, z=227982, k=110 digits : 0030 ... 7576
n=23, x=13019, y=19919, z=24979, k=89 digits : 0002 ... 0759
n=23, x=14511, y=18586, z=54097, k=96 digits : 4999 ... 7403
n=23, x=14537, y=14908, z=76605, k=100 digits : 1311 ... 1757

7.11 n=29

7.11.1 n=29, x=2 to 58

n=29, x=2, y=7729, z=36027, k=124 digits : 2491 ... 9991
n=29, x=3, y=295, z=1858, k=89 digits : 0003 ... 9799
n=29, x=3, y=349, z=472, k=72 digits : 7085 ... 9540
n=29, x=3, y=1741, z=5584, k=102 digits : 0015 ... 9685
n=29, x=3, y=2792, z=3539, k=96 digits : 2787 ... 0186
n=29, x=3, y=2902, z=4897, k=100 digits : 2389 ... 6471
n=29, x=3, y=3068, z=28967, k=121 digits : 0009 ... 1447
n=29, x=3, y=5900, z=9803, k=108 digits : 3236 ... 6702
n=29, x=3, y=6077, z=53048, k=129 digits : 0001 ... 5336
n=29, x=4, y=8529, z=12349, k=111 digits : 0107 ... 9379

n=29, x=5, y=118, z=523, k=74 digits : 0022 ... 8031
n=29, x=5, y=523, z=5108, k=101 digits : 0002 ... 6372
n=29, x=5, y=523, z=2563, k=93 digits : 0001 ... 6743
n=29, x=5, y=944, z=3029, k=94 digits : 0063 ... 3843
n=29, x=5, y=3138, z=6503, k=103 digits : 0372 ... 4723
n=29, x=5, y=4003, z=9853, k=108 digits : 3300 ... 3095
n=29, x=5, y=4188, z=12413, k=111 digits : 0203 ... 8970
n=29, x=7, y=233, z=590, k=75 digits : 0235 ... 5896
n=29, x=7, y=708, z=2563, k=92 digits : 5621 ... 6271
n=29, x=7, y=1973, z=10485, k=109 digits : 0002 ... 7543
n=29, x=7, y=3831, z=5959, k=102 digits : 0018 ... 9887
n=29, x=11, y=1829, z=6614, k=103 digits : 0467 ... 8264
n=29, x=11, y=2089, z=7721, k=105 digits : 0003 ... 5359
n=29, x=11, y=3482, z=13811, k=112 digits : 2203 ... 7044
n=29, x=12, y=233, z=1745, k=88 digits : 2107 ... 4006
n=29, x=13, y=177, z=1165, k=83 digits : 0312 ... 9031
n=29, x=14, y=177, z=233, k=63 digits : 0779 ... 9268
n=29, x=17, y=5230, z=21057, k=117 digits : 0001 ... 2328
n=29, x=20, y=531, z=1451, k=85 digits : 0003 ... 0294
n=29, x=27, y=413, z=2330, k=91 digits : 0173 ... 4028
n=29, x=29, y=233, z=2089, k=90 digits : 0013 ... 0319
n=29, x=40, y=59, z=699, k=77 digits : 0001 ... 9344
n=29, x=45, y=118, z=523, k=73 digits : 0002 ... 5059
n=29, x=46, y=2843, z=10617, k=108 digits : 4088 ... 6523
n=29, x=47, y=5515, z=6267, k=101 digits : 0007 ... 3383
n=29, x=49, y=1567, z=6614, k=103 digits : 0122 ... 2824
n=29, x=49, y=7892, z=8529, k=105 digits : 0002 ... 0264
n=29, x=58, y=233, z=2089, k=89 digits : 0006 ... 5568

7.11.2 n=29, x=59

Prime x,n, $2mn+1$ form, x=2.11+1, n=29, m=1

n	x	y	z	k
29	59	160	699	76 digits
29	59	261	1103	82 digits
29	59	841	5801	101 digits
29	59	929	4427	98 digits
29	59	1552	6267	102 digits
29	59	5046	7193	103 digits : 0330 ... 4199
29	59	6335	57074	128 digits : 4052 ... 8631
29	59	6336 to 9999	none	

7.11.3 n=29, x=60 to 9999

n=29, x=66, y=1103, z=1631, k=86 digits : 0012 ... 4504
n=29, x=81, y=4931, z=33119, k=121 digits : 0009 ... 9143

n=29, x=88, y=2089, z=7721, k=104 digits : 3894 ... 1392
n=29, x=101, y=699, z=1103, k=81 digits : 0002 ... 3239
n=29, x=103, y=9320, z=75343, k=131 digits : 0375 ... 1317
n=29, x=116, y=233, z=2089, k=89 digits : 0003 ... 6560
n=29, x=118, y=9473, z=28537, k=119 digits : 0504 ... 9172
n=29, x=134, y=1165, z=3309, k=94 digits : 0022 ... 8080
n=29, x=177, y=478, z=2089, k=89 digits : 0001 ... 5396
n=29, x=177, y=7223, z=37160, k=122 digits : 0071 ... 5965
n=29, x=203, y=7193, z=20591, k=115 digits : 0415 ... 5015
n=29, x=232, y=233, z=2089, k=89 digits : 0001 ... 0136
n=29, x=233, y=435, z=1103, k=81 digits : 0001 ... 0567
n=29, x=233, y=464, z=2089, k=88 digits : 8402 ... 6404
n=29, x=233, y=493, z=4177, k=97 digits : 0002 ... 4207
n=29, x=233, y=769, z=3309, k=94 digits : 0019 ... 8199
n=29, x=233, y=928, z=2089, k=88 digits : 4201 ... 7418
n=29, x=233, y=936, z=7721, k=104 digits : 3282 ... 1144
n=29, x=236, y=7087, z=35671, k=122 digits : 0017 ... 8454
n=29, x=249, y=8368, z=28537, k=119 digits : 0270 ... 7240
n=29, x=261, y=2089, z=4177, k=96 digits : 4442 ... 4639
n=29, x=295, y=932, z=2787, k=92 digits : 1058 ... 7079
n=29, x=301, y=699, z=5515, k=100 digits : 2755 ... 7975
n=29, x=305, y=2089, z=3309, k=93 digits : 0005 ... 5991
n=29, x=349, y=2124, z=11833, k=109 digits : 0001 ... 3085
n=29, x=443, y=4177, z=7721, k=103 digits : 0386 ... 4671
n=29, x=446, y=3309, z=10445, k=107 digits : 0229 ... 1216
n=29, x=493, y=929, z=10789, k=108 digits : 1830 ... 9559
n=29, x=493, y=1103, z=2089, k=88 digits : 1670 ... 1143
n=29, x=523, y=2649, z=24364, k=117 digits : 0004 ... 2499
n=29, x=598, y=2843, z=10617, k=107 digits : 0314 ... 8407
n=29, x=704, y=2089, z=7721, k=103 digits : 0486 ... 3928
n=29, x=719, y=9573, z=16763, k=112 digits : 2780 ... 5951
n=29, x=723, y=1973, z=11372, k=108 digits : 2565 ... 4782
n=29, x=767, y=6631, z=31851, k=120 digits : 2404 ... 1479
n=29, x=929, y=1451, z=4012, k=95 digits : 0581 ... 3209
n=29, x=929, y=6903, z=63173, k=128 digits : 4053 ... 6911
n=29, x=1003, y=1277, z=3946, k=95 digits : 0384 ... 0036
n=29, x=1103, y=1291, z=6267, k=101 digits : 0001 ... 4583
n=29, x=1103, y=1690, z=14623, k=111 digits : 0224 ... 6088
n=29, x=1103, y=3161, z=13747, k=110 digits : 0021 ... 9343
n=29, x=1103, y=3517, z=29239, k=119 digits : 0287 ... 0511
n=29, x=1103, y=5605, z=36348, k=121 digits : 0007 ... 5859
n=29, x=1357, y=3539, z=15844, k=111 digits : 0821 ... 0024
n=29, x=1451, y=2183, z=24058, k=117 digits : 0001 ... 9191
n=29, x=1534, y=1741, z=27845, k=119 digits : 0106 ... 8180
n=29, x=1549, y=3539, z=23676, k=116 digits : 5516 ... 2088
n=29, x=1631, y=2792, z=3191, k=92 digits : 2769 ... 0699
n=29, x=1870, y=6267, z=13747, k=109 digits : 0006 ... 8484
n=29, x=1973, y=2787, z=4130, k=95 digits : 0320 ... 4492

n=29, x=1973, y=3115, z=42468, k=123 digits : 0626 ... 2888
n=29, x=2089, y=2447, z=41241, k=123 digits : 0331 ... 1295
n=29, x=2089, y=9788, z=41241, k=122 digits : 0082 ... 9892
n=29, x=2242, y=4701, z=49237, k=125 digits : 0002 ... 8076
n=29, x=2419, y=3716, z=12531, k=108 digits : 6164 ... 3344
n=29, x=2443, y=4661, z=66828, k=129 digits : 0001 ... 0096
n=29, x=2494, y=6091, z=28537, k=118 digits : 0037 ... 2219
n=29, x=2615, y=8529, z=46964, k=124 digits : 2891 ... 1666
n=29, x=5243, y=9573, z=62003, k=127 digits : 0306 ... 9031
n=29, x=5801, y=6799, z=10325, k=105 digits : 0006 ... 6207
n=29, x=6323, y=6757, z=6961, k=100 digits : 4754 ... 5751

7.11.4 n=29, x=10000 to 19999

There are no solutions for n=29, x=10000 to 19999, y=x+1.

7.12 n=31

7.12.1 n=31, x=2 to 9999

n=31, x=3, y=1427, z=4043, k=105 digits : 0003 ... 4639
n=31, x=8, y=5209, z=14617, k=121 digits : 0002 ... 2968
n=31, x=9, y=1492, z=4093, k=105 digits : 0001 ... 1535
n=31, x=9, y=6263, z=60956, k=139 digits : 0630 ... 2080
n=31, x=11, y=1492, z=3351, k=102 digits : 0034 ... 4561
n=31, x=18, y=2177, z=20465, k=125 digits : 0005 ... 2408
n=31, x=19, y=1117, z=10294, k=117 digits : 0001 ... 6616
n=31, x=23, y=3732, z=6515, k=110 digits : 0030 ... 8557
n=31, x=23, y=7444, z=38931, k=133 digits : 0002 ... 4805
n=31, x=56, y=4839, z=15815, k=121 digits : 0003 ... 3228
n=31, x=62, y=311, z=1117, k=88 digits : 1433 ... 2011
n=31, x=73, y=8936, z=62153, k=138 digits : 0097 ... 5776
n=31, x=128, y=2049, z=7937, k=112 digits : 3723 ... 2178
n=31, x=311, y=2976, z=21143, k=124 digits : 6146 ... 3865
n=31, x=551, y=1861, z=15636, k=120 digits : 6499 ... 6969
n=31, x=613, y=1366, z=6883, k=110 digits : 0016 ... 0051
n=31, x=683, y=2984, z=3907, k=102 digits : 0027 ... 4073
n=31, x=1117, y=1427, z=2238, k=95 digits : 0196 ... 7768
n=31, x=1135, y=9859, z=37099, k=131 digits : 0107 ... 7159
n=31, x=1240, y=9859, z=39619, k=131 digits : 0707 ... 5864
n=31, x=2364, y=5581, z=43405, k=133 digits : 0001 ... 6704

7.12.2 n=31, x=10000 to 19999

There are no solutions for n=31, x=10000 to 19999, y=x+1.

7.13 n=37

7.13.1 n=37, x=2 to 9999

n=37, x=2, y=4443, z=7805, k=137 digits : 0001 ... 6527
n=37, x=4, y=1481, z=6705, k=134 digits : 0095 ... 7154
n=37, x=5, y=1043, z=8363, k=138 digits : 0030 ... 6967
n=37, x=5, y=1341, z=1481, k=111 digits : 0200 ... 8751
n=37, x=7, y=6407, z=7919, k=136 digits : 5014 ... 6175
n=37, x=8, y=223, z=447, k=93 digits : 0001 ... 7132
n=37, x=8, y=4683, z=18131, k=149 digits : 0005 ... 4085
n=37, x=9, y=149, z=446, k=93 digits : 0001 ... 0643
n=37, x=11, y=8661, z=12434, k=143 digits : 0267 ... 9568
n=37, x=13, y=2372, z=7897, k=136 digits : 6600 ... 1851
n=37, x=13, y=2980, z=13913, k=145 digits : 0003 ... 4115
n=37, x=14, y=223, z=447, k=92 digits : 8253 ... 6720
n=37, x=29, y=669, z=1481, k=110 digits : 0071 ... 1823
n=37, x=35, y=669, z=2384, k=118 digits : 0016 ... 6908
n=37, x=41, y=7919, z=61055, k=167 digits : 0594 ... 4399
n=37, x=57, y=223, z=1192, k=107 digits : 0438 ... 4891
n=37, x=61, y=6143, z=31092, k=157 digits : 0001 ... 2843
n=37, x=64, y=2235, z=4219, k=126 digits : 0022 ... 2931
n=37, x=67, y=3998, z=40379, k=161 digits : 0002 ... 8836
n=37, x=74, y=593, z=1777, k=113 digits : 0002 ... 3920
n=37, x=80, y=1043, z=3923, k=125 digits : 0002 ... 1428
n=37, x=98, y=1777, z=3345, k=122 digits : 0043 ... 2000
n=37, x=111, y=3923, z=7919, k=135 digits : 0516 ... 6495
n=37, x=121, y=8363, z=45147, k=162 digits : 0036 ... 0423
n=37, x=149, y=176, z=2965, k=121 digits : 0003 ... 8960
n=37, x=149, y=1295, z=7919, k=136 digits : 1165 ... 9567
n=37, x=149, y=1369, z=1481, k=109 digits : 0006 ... 5343
n=37, x=162, y=1999, z=7993, k=135 digits : 0971 ... 4003
n=37, x=163, y=2007, z=12439, k=142 digits : 0078 ... 9311
n=37, x=172, y=593, z=3345, k=122 digits : 0074 ... 2496
n=37, x=191, y=1043, z=7919, k=136 digits : 1129 ... 9295
n=37, x=192, y=1561, z=4441, k=126 digits : 0068 ... 7099
n=37, x=222, y=3331, z=17539, k=147 digits : 0822 ... 8272
n=37, x=223, y=793, z=5774, k=131 digits : 0146 ... 6948
n=37, x=223, y=1186, z=6143, k=131 digits : 0910 ... 8736
n=37, x=223, y=1277, z=5331, k=129 digits : 0005 ... 2911
n=37, x=223, y=7301, z=67941, k=168 digits : 5559 ... 2439
n=37, x=234, y=7993, z=16651, k=146 digits : 0050 ... 0447
n=37, x=239, y=2682, z=20129, k=150 digits : 0013 ... 8819
n=37, x=246, y=7919, z=61055, k=166 digits : 0099 ... 4536
n=37, x=252, y=7993, z=17317, k=147 digits : 0190 ... 3701
n=37, x=253, y=6407, z=61055, k=167 digits : 0119 ... 9471
n=37, x=277, y=669, z=1777, k=112 digits : 5258 ... 4911
n=37, x=396, y=1999, z=16651, k=147 digits : 0118 ... 9309
n=37, x=414, y=1999, z=17317, k=147 digits : 0464 ... 6727

n=37, x=447, y=3257, z=10364, k=139 digits : 0248 ... 5610
n=37, x=593, y=892, z=1341, k=107 digits : 0731 ... 9141
n=37, x=666, y=1481, z=20129, k=149 digits : 0008 ... 0588
n=37, x=669, y=1937, z=18206, k=148 digits : 1799 ... 1835
n=37, x=745, y=1911, z=12211, k=141 digits : 0009 ... 1855
n=37, x=892, y=6143, z=8895, k=136 digits : 2694 ... 1344
n=37, x=1028, y=4443, z=33227, k=157 digits : 0001 ... 1052
n=37, x=1476, y=7919, z=61055, k=166 digits : 0016 ... 2548
n=37, x=1561, y=6109, z=67366, k=167 digits : 0699 ... 2899
n=37, x=4967, y=8885, z=88357, k=171 digits : 0262 ... 9527
n=37, x=4967, y=8885, z=88357, k=171 digits : 0262 ... 9527
n=37, x=7919, y=8856, z=61055, k=165 digits : 0002 ... 2110
n=37, x=8882, y=9103, z=92135, k=171 digits : 0648 ... 8988

7.13.2 n=37, x=10000 to 19999

There are no solutions for n=37, x=10000 to 19999, y=x+1.

7.14 n=41

7.14.1 n=41, x=2 to 82

n=41, x=2, y=415, z=2297, k=132 digits : 3367 ... 2747
n=41, x=3, y=166, z=739, k=113 digits : 0001 ... 7360
n=41, x=3, y=1559, z=5987, k=148 digits : 2620 ... 1175
n=41, x=3, y=4316, z=13367, k=161 digits : 0008 ... 5703
n=41, x=4, y=821, z=1245, k=121 digits : 0001 ... 9210
n=41, x=4, y=5395, z=16319, k=165 digits : 0001 ... 5327
n=41, x=4, y=6971, z=20095, k=168 digits : 4766 ... 3205
n=41, x=4, y=8367, z=68143, k=189 digits : 0006 ... 5728
n=41, x=5, y=9031, z=35436, k=178 digits : 0021 ... 6981
n=41, x=9, y=1826, z=5003, k=144 digits : 5668 ... 2859
n=41, x=11, y=332, z=739, k=112 digits : 1524 ... 4482
n=41, x=11, y=2241, z=9677, k=156 digits : 1090 ... 0175
n=41, x=16, y=8549, z=18453, k=166 digits : 0032 ... 3419
n=41, x=17, y=415, z=2217, k=130 digits : 0095 ... 7295
n=41, x=24, y=581, z=2789, k=134 digits : 0047 ... 6524
n=41, x=67, y=332, z=2543, k=132 digits : 7355 ... 1157

7.14.2 n=41, x=83

Prime x,n, $2mn+1$ form, x=2.41+1, n=41, m=1

n	x	y	z	k
41	83	361	3693	139 digits
41	83	552	6155	147 digits
41	83	1723	6151	147 digits

41	83	3321	13367	160 digits : 3990 ... 6423
41	83	9677	88568	192 digits : 9688 ... 2716
41	83	9678 to 9999	none	

7.14.3 n=41, x=84 to 9999

n=41, x=112, y=7387, z=17467, k=164 digits : 5902 ... 5466
n=41, x=137, y=8367, z=21656, k=168 digits : 2311 ... 6573
n=41, x=166, y=739, z=2953, k=134 digits : 0052 ... 3059
n=41, x=239, y=1660, z=16239, k=163 digits : 0666 ... 9600
n=41, x=415, y=4924, z=15599, k=162 digits : 0025 ... 1020
n=41, x=556, y=8715, z=67651, k=187 digits : 0335 ... 0538
n=41, x=664, y=2543, z=28047, k=172 digits : 4874 ... 0860
n=41, x=664, y=3671, z=33703, k=175 digits : 0524 ... 5774
n=41, x=712, y=2905, z=28537, k=172 digits : 7956 ... 9860
n=41, x=739, y=996, z=1723, k=124 digits : 3841 ... 3174
n=41, x=984, y=6151, z=68143, k=187 digits : 0358 ... 9759
n=41, x=3735, y=8693, z=10673, k=154 digits : 0041 ... 8567
n=41, x=4105, y=5146, z=45211, k=179 digits : 0767 ... 8783
n=41, x=5003, y=8693, z=73504, k=188 digits : 1032 ... 9388

7.14.4 n=41, x=10000 to 19999

n=41, x=10169, y=11896, z=200505, k=205 digits : 0001 ... 3408

7.15 n=43

7.15.1 n=43, x=2 to 9999

n=43, x=2, y=6451, z=41347, k=190 digits : 0060 ... 6456
n=43, x=3, y=7576, z=30283, k=184 digits : 7141 ... 1041
n=43, x=4, y=431, z=2595, k=141 digits : 0001 ... 1839
n=43, x=4, y=2237, z=10581, k=166 digits : 0011 ... 1698
n=43, x=5, y=692, z=4817, k=152 digits : 1372 ... 6725
n=43, x=8, y=3879, z=23759, k=180 digits : 1963 ... 1831
n=43, x=9, y=862, z=4903, k=152 digits : 1287 ... 5055
n=43, x=11, y=692, z=1979, k=135 digits : 0370 ... 5140
n=43, x=12, y=947, z=1211, k=126 digits : 0027 ... 9570
n=43, x=12, y=2249, z=8189, k=160 digits : 8404 ... 4981
n=43, x=20, y=5017, z=10837, k=165 digits : 0002 ... 1313
n=43, x=21, y=5419, z=46957, k=192 digits : 1429 ... 8511
n=43, x=31, y=1033, z=3017, k=142 digits : 0043 ... 3535
n=43, x=43, y=173, z=431, k=107 digits : 0597 ... 5063
n=43, x=70, y=173, z=1293, k=127 digits : 0401 ... 7888
n=43, x=89, y=9806, z=14879, k=170 digits : 0020 ... 8539

n=43, x=121, y=862, z=4129, k=147 digits : 0703 ... 0100
n=43, x=173, y=947, z=3017, k=141 digits : 0008 ... 9799
n=43, x=173, y=947, z=6196, k=155 digits : 0113 ... 4296
n=43, x=196, y=4647, z=9547, k=162 digits : 0015 ... 2451
n=43, x=229, y=8171, z=74352, k=199 digits : 0209 ... 8681
n=43, x=431, y=1161, z=9719, k=162 digits : 0060 ... 7503
n=43, x=431, y=2322, z=9719, k=162 digits : 0030 ... 5740
n=43, x=431, y=4644, z=9719, k=162 digits : 0015 ... 3174
n=43, x=519, y=575, z=9719, k=163 digits : 0101 ... 3663
n=43, x=692, y=1291, z=8259, k=159 digits : 0363 ... 6690
n=43, x=865, y=9719, z=65079, k=196 digits : 1737 ... 2527
n=43, x=1291, y=3698, z=10837, k=163 digits : 0612 ... 6115
n=43, x=1903, y=5172, z=46015, k=189 digits : 0007 ... 7740
n=43, x=3698, y=4903, z=36809, k=185 digits : 0003 ... 9335
n=43, x=7576, y=8429, z=8605, k=158 digits : 0016 ... 4358

7.15.2 n=43, x=10000 to 19999

There are no solutions for n=43, x=10000 to 19999, y=x+1.

7.16 n=47

7.16.1 n=47, x=2 to 9999

n=47, x=4, y=3387, z=14947, k=188 digits : 7899 ... 4966
n=47, x=4, y=8561, z=13913, k=187 digits : 0115 ... 0246
n=47, x=4, y=8837, z=18049, k=192 digits : 1772 ... 8441
n=47, x=5, y=283, z=1318, k=141 digits : 0002 ... 7344
n=47, x=5, y=8773, z=86898, k=223 digits : 0356 ... 1875
n=47, x=7, y=2830, z=5077, k=167 digits : 0144 ... 4023
n=47, x=7, y=3387, z=9778, k=180 digits : 1501 ... 8143
n=47, x=9, y=5171, z=16457, k=190 digits : 0019 ... 4471
n=47, x=12, y=1693, z=13165, k=186 digits : 0015 ... 5764
n=47, x=47, y=2069, z=8461, k=176 digits : 4715 ... 1303
n=47, x=47, y=3761, z=4513, k=163 digits : 0720 ... 4223
n=47, x=48, y=941, z=6509, k=171 digits : 0584 ... 8668
n=47, x=69, y=283, z=1318, k=140 digits : 1680 ... 6656
n=47, x=82, y=7333, z=67685, k=217 digits : 0002 ... 6128
n=47, x=119, y=566, z=2351, k=151 digits : 0177 ... 2764
n=47, x=182, y=849, z=3761, k=160 digits : 1883 ... 9648
n=47, x=188, y=1129, z=4889, k=165 digits : 0002 ... 6876
n=47, x=229, y=849, z=4513, k=163 digits : 0655 ... 1783
n=47, x=283, y=517, z=2351, k=150 digits : 0081 ... 7991
n=47, x=283, y=3314, z=13539, k=185 digits : 0001 ... 9436
n=47, x=376, y=659, z=6299, k=170 digits : 0023 ... 3169
n=47, x=635, y=4513, z=7053, k=171 digits : 0369 ... 7423

n=47, x=793, y=9784, z=16921, k=188 digits : 4147 ... 7120
n=47, x=849, y=916, z=4513, k=163 digits : 0163 ... 7436
n=47, x=941, y=2539, z=14106, k=185 digits : 0003 ... 1604
n=47, x=1013, y=2351, z=32741, k=202 digits : 0020 ... 5911
n=47, x=1081, y=2351, z=4513, k=162 digits : 0050 ... 3535
n=47, x=1357, y=9784, z=21997, k=193 digits : 0004 ... 3716
n=47, x=1493, y=4513, z=16457, k=188 digits : 1329 ... 9463
n=47, x=1922, y=4513, z=35265, k=203 digits : 0173 ... 1648
n=47, x=1969, y=3761, z=35265, k=203 digits : 0203 ... 9199
n=47, x=2209, y=6299, z=21433, k=193 digits : 0001 ... 9303
n=47, x=2351, y=2797, z=13539, k=184 digits : 1718 ... 3855
n=47, x=2351, y=3379, z=56415, k=212 digits : 4614 ... 0999
n=47, x=2351, y=3655, z=31591, k=201 digits : 0001 ... 3231
n=47, x=2446, y=6517, z=21997, k=193 digits : 0003 ... 2156

7.16.2 n=47, x=10000 to 19999

n=47, x=11336, y=16457, z=186497, k=235 digits : 0151 ... 9521
n=47, x=13165, y=14676, z=180961, k=234 digits : 0036 ... 2223

7.17 n=53

7.17.1 n=53, x=2 to 9999

n=53, x=2, y=2889, z=4241, k=185 digits : 0007 ... 6420
n=53, x=2, y=3959, z=21943, k=222 digits : 0070 ... 0528
n=53, x=2, y=5029, z=9543, k=203 digits : 0873 ... 9523
n=53, x=2, y=5091, z=49541, k=241 digits : 0001 ... 9359
n=53, x=3, y=107, z=743, k=147 digits : 0609 ... 7471
n=53, x=3, y=1061, z=2996, k=178 digits : 0018 ... 4874
n=53, x=5, y=743, z=4708, k=188 digits : 2615 ... 6771
n=53, x=8, y=535, z=3183, k=179 digits : 0328 ... 4335
n=53, x=8, y=1697, z=16585, k=216 digits : 1960 ... 3281
n=53, x=8, y=6999, z=68159, k=247 digits : 0393 ... 0889
n=53, x=9, y=7597, z=58726, k=244 digits : 1394 ... 1315
n=53, x=11, y=321, z=2972, k=178 digits : 0011 ... 9405
n=53, x=14, y=1819, z=3499, k=180 digits : 7570 ... 7544
n=53, x=19, y=1061, z=9630, k=203 digits : 0698 ... 0948
n=53, x=20, y=321, z=1061, k=154 digits : 0033 ... 1661
n=53, x=32, y=2333, z=23005, k=222 digits : 0087 ... 7006
n=53, x=35, y=856, z=5091, k=189 digits : 0001 ... 6888
n=53, x=53, y=107, z=743, k=146 digits : 0034 ... 0071
n=53, x=67, y=2333, z=6420, k=193 digits : 0006 ... 4840
n=53, x=96, y=1697, z=17441, k=216 digits : 2237 ... 0724
n=53, x=107, y=4744, z=19083, k=217 digits : 0007 ... 7348
n=53, x=107, y=5359, z=48338, k=238 digits : 0066 ... 3543
n=53, x=121, y=9737, z=97946, k=254 digits : 0028 ... 3659
n=53, x=214, y=7985, z=24169, k=222 digits : 0049 ... 8484

n=53, x=238, y=743, z=9549, k=202 digits : 0051 ... 1783
n=53, x=321, y=1510, z=6361, k=193 digits : 0001 ... 8612
n=53, x=321, y=9332, z=27461, k=225 digits : 0002 ... 2989
n=53, x=743, y=2969, z=13696, k=209 digits : 0005 ... 9187
n=53, x=1697, y=2809, z=25229, k=223 digits : 0166 ... 6199
n=53, x=2333, y=2809, z=23321, k=221 digits : 0002 ... 7487
n=53, x=4666, y=5091, z=94909, k=252 digits : 2781 ... 8503
n=53, x=4731, y=6679, z=44527, k=235 digits : 0169 ... 7119
n=53, x=5049, y=6361, z=46753, k=236 digits : 2105 ... 1279
n=53, x=6679, y=7844, z=69431, k=245 digits : 0001 ... 7928

7.17.2 n=53, x=10000 to 19999

n=53, x=12587, y=19083, z=69431, k=244 digits : 2399 ... 2031

7.18 n=59

7.18.1 n=59, x=2 to 9999

n=59, x=2, y=2243, z=6373, k=218 digits : 0010 ... 1439
n=59, x=3, y=5789, z=20534, k=246 digits : 0076 ... 2096
n=59, x=7, y=8499, z=37171, k=261 digits : 0001 ... 3463
n=59, x=11, y=1889, z=3187, k=199 digits : 0756 ... 7351
n=59, x=11, y=5789, z=55465, k=271 digits : 0223 ... 9255
n=59, x=12, y=2833, z=5905, k=215 digits : 0158 ... 7776
n=59, x=28, y=8499, z=37171, k=260 digits : 4958 ... 0320
n=59, x=41, y=5672, z=34249, k=258 digits : 0043 ... 9660
n=59, x=71, y=9217, z=42516, k=263 digits : 0436 ... 3646
n=59, x=112, y=8499, z=37171, k=260 digits : 1239 ... 1808
n=59, x=139, y=1889, z=4252, k=206 digits : 0010 ... 9135
n=59, x=192, y=2833, z=5905, k=213 digits : 0009 ... 2320
n=59, x=448, y=8499, z=37171, k=259 digits : 0309 ... 2160
n=59, x=664, y=3541, z=14165, k=235 digits : 0250 ... 6368
n=59, x=827, y=2833, z=4252, k=205 digits : 0001 ... 5479
n=59, x=1063, y=3481, z=7907, k=220 digits : 3284 ... 5495
n=59, x=1792, y=8499, z=37171, k=258 digits : 0077 ... 6528
n=59, x=7168, y=8499, z=37171, k=258 digits : 0019 ... 6700

7.18.2 n=59, x=10000 to 9999

n=59, x=10583, y=10623, z=179951, k=297 digits : 0005 ... 2063
n=59, x=10623, y=10952, z=185855, k=298 digits : 0035 ... 3360

7.19 n=61

7.19.1 n=61, x=2 to 19999

n=61, x=2, y=1709, z=15373, k=248 digits : 4696 ... 5408
n=61, x=2, y=7687, z=12081, k=241 digits : 0005 ... 2243
n=61, x=2, y=9883, z=10995, k=239 digits : 0149 ... 8892
n=61, x=2, y=9883, z=23795, k=259 digits : 0196 ... 9612
n=61, x=2, y=15373, z=30053, k=265 digits : 0001 ... 1356
n=61, x=3, y=4885, z=34498, k=269 digits : 0001 ... 4428
n=61, x=3, y=5857, z=19540, k=254 digits : 0016 ... 6381
n=61, x=3, y=11714, z=81131, k=291 digits : 0101 ... 2132
n=61, x=4, y=7323, z=15739, k=248 digits : 2248 ... 3624
n=61, x=4, y=10617, z=15781, k=248 digits : 1819 ... 8195
n=61, x=6, y=5857, z=9529, k=235 digits : 0157 ... 9012
n=61, x=7, y=977, z=2202, k=197 digits : 0005 ... 6956
n=61, x=8, y=9155, z=11963, k=240 digits : 6392 ... 4899
n=61, x=11, y=8545, z=34131, k=268 digits : 1037 ... 9831
n=61, x=16, y=19791, z=145087, k=305 digits : 0001 ... 9115
n=61, x=17, y=4404, z=8297, k=231 digits : 0182 ... 7606
n=61, x=25, y=3539, z=24189, k=259 digits : 0117 ... 8191
n=61, x=26, y=7687, z=12081, k=240 digits : 4221 ... 5519
n=61, x=29, y=4771, z=7816, k=229 digits : 0002 ... 3064
n=61, x=42, y=1709, z=12461, k=241 digits : 0007 ... 9920
n=61, x=61, y=733, z=1709, k=190 digits : 0020 ... 4295
n=61, x=63, y=977, z=7340, k=228 digits : 1420 ... 1884
n=61, x=81, y=5003, z=15047, k=246 digits : 0010 ... 3703
n=61, x=93, y=2936, z=12701, k=241 digits : 0006 ... 4264
n=61, x=116, y=5505, z=31721, k=265 digits : 0001 ... 5946
n=61, x=116, y=17569, z=147141, k=304 digits : 5686 ... 5629
n=61, x=133, y=13911, z=114193, k=298 digits : 0015 ... 1911
n=61, x=185, y=4404, z=9029, k=232 digits : 2675 ... 6775
n=61, x=196, y=4637, z=6597, k=224 digits : 1593 ... 2706
n=61, x=332, y=8053, z=12701, k=240 digits : 6355 ... 6646
n=61, x=338, y=7687, z=12081, k=239 digits : 0324 ... 6971
n=61, x=361, y=8054, z=32241, k=265 digits : 0001 ... 0844
n=61, x=367, y=3721, z=4027, k=211 digits : 0144 ... 7551
n=61, x=367, y=18911, z=176799, k=309 digits : 0001 ... 3967
n=61, x=535, y=12081, z=19036, k=250 digits : 0092 ... 4598
n=61, x=575, y=734, z=4759, k=216 digits : 1060 ... 6692
n=61, x=733, y=4100, z=41733, k=271 digits : 0563 ... 3490
n=61, x=733, y=11163, z=17203, k=248 digits : 1672 ... 7111
n=61, x=873, y=4759, z=16108, k=246 digits : 0063 ... 3696
n=61, x=873, y=7687, z=19036, k=250 digits : 0088 ... 8916
n=61, x=1267, y=14277, z=30748, k=263 digits : 0102 ... 5722
n=61, x=1830, y=4027, z=27817, k=260 digits : 6182 ... 6727
n=61, x=2569, y=6633, z=55633, k=278 digits : 0030 ... 8591
n=61, x=4027, y=11163, z=15739, k=245 digits : 0001 ... 0391
n=61, x=4759, y=8193, z=111268, k=296 digits : 1553 ... 5326

n=61, x=4955, y=19036, z=83451, k=288 digits : 2047 ... 8888
n=61, x=5828, y=7687, z=83451, k=288 digits : 4311 ... 2501
n=61, x=6973, y=17203, z=172756, k=307 digits : 0146 ... 5820
n=61, x=14884, y=15373, z=249857, k=316 digits : 3176 ... 2925

7.20 n=67

7.20.1 n=67, x=2 to 19999

n=67, x=3, y=2690, z=3083, k=227 digits : 0231 ... 2044
n=67, x=3, y=5380, z=8443, k=255 digits : 0872 ... 0586
n=67, x=4, y=9385, z=11029, k=263 digits : 0170 ... 2705
n=67, x=4, y=9953, z=23721, k=285 digits : 0001 ... 1510
n=67, x=15, y=3497, z=12872, k=267 digits : 0328 ... 8125
n=67, x=19, y=9649, z=32563, k=293 digits : 0003 ... 5143
n=67, x=21, y=4304, z=16085, k=273 digits : 0004 ... 1940
n=67, x=44, y=2011, z=4827, k=239 digits : 0149 ... 9376
n=67, x=47, y=3217, z=24132, k=285 digits : 0001 ... 7024
n=67, x=48, y=4573, z=18493, k=277 digits : 0001 ... 9654
n=67, x=57, y=1609, z=4573, k=237 digits : 0004 ... 8415
n=67, x=67, y=269, z=3217, k=228 digits : 1721 ... 8471
n=67, x=83, y=19699, z=41279, k=299 digits : 0265 ... 8431
n=67, x=96, y=1609, z=9385, k=257 digits : 0009 ... 5331
n=67, x=107, y=2152, z=4827, k=238 digits : 0057 ... 7794
n=67, x=112, y=3083, z=19099, k=278 digits : 0010 ... 1341
n=67, x=268, y=269, z=1609, k=207 digits : 0595 ... 2591
n=67, x=269, y=5226, z=26399, k=286 digits : 0047 ... 8139
n=67, x=269, y=17956, z=28409, k=288 digits : 1754 ... 7369
n=67, x=1356, y=11263, z=22111, k=280 digits : 3632 ... 9096
n=67, x=1609, y=4958, z=26399, k=285 digits : 0008 ... 3851
n=67, x=1609, y=7263, z=12868, k=265 digits : 0001 ... 5966
n=67, x=1609, y=19146, z=154777, k=336 digits : 1076 ... 2856
n=67, x=2653, y=12872, z=79197, k=316 digits : 6044 ... 4944
n=67, x=3217, y=6456, z=72913, k=314 digits : 0042 ... 6808
n=67, x=5227, y=6725, z=11927, k=262 digits : 0031 ... 8759
n=67, x=11256, y=12329, z=169913, k=338 digits : 0011 ... 8506

7.21 n=71

7.21.1 n=71, x=2 to 19999

n=71, x=2, y=13495, z=42207, k=320 digits : 2216 ... 1724
n=71, x=3, y=4261, z=20452, k=298 digits : 0044 ... 4790
n=71, x=3, y=10226, z=44021, k=321 digits : 0003 ... 1071
n=71, x=3, y=14917, z=137035, k=355 digits : 0846 ... 2527
n=71, x=5, y=5116, z=5541, k=258 digits : 0043 ... 0360
n=71, x=5, y=7677, z=29417, k=309 digits : 0001 ... 2063
n=71, x=7, y=17048, z=76695, k=337 digits : 0007 ... 7514

n=71, x=12, y=569, z=2273, k=232 digits : 1341 ... 3330
n=71, x=13, y=1847, z=13495, k=285 digits : 0005 ... 5783
n=71, x=15, y=853, z=5113, k=256 digits : 3164 ... 6799
n=71, x=16, y=4261, z=7397, k=267 digits : 0100 ... 0908
n=71, x=20, y=10811, z=40471, k=318 digits : 0014 ... 1253
n=71, x=27, y=2131, z=17467, k=293 digits : 0001 ... 0751
n=71, x=34, y=19915, z=24709, k=302 digits : 0046 ... 4995
n=71, x=37, y=15363, z=70345, k=334 digits : 0035 ... 5495
n=71, x=43, y=1138, z=2557, k=234 digits : 0071 ... 4623
n=71, x=51, y=1279, z=5971, k=260 digits : 3227 ... 2199
n=71, x=67, y=9941, z=38349, k=316 digits : 1094 ... 5375
n=71, x=79, y=2131, z=8530, k=270 digits : 0087 ... 6447
n=71, x=79, y=14225, z=109104, k=347 digits : 0396 ... 1014
n=71, x=119, y=19597, z=52917, k=325 digits : 0001 ... 5431
n=71, x=142, y=853, z=2131, k=228 digits : 8271 ... 1271
n=71, x=162, y=8953, z=23857, k=301 digits : 0001 ... 6804
n=71, x=431, y=2845, z=26981, k=305 digits : 0001 ... 7015
n=71, x=569, y=853, z=2558, k=233 digits : 0007 ... 0671
n=71, x=1207, y=5113, z=19597, k=294 digits : 0046 ... 0999
n=71, x=1279, y=9235, z=57659, k=327 digits : 0154 ... 5063
n=71, x=1704, y=17041, z=49417, k=322 digits : 0012 ... 0901
n=71, x=1847, y=4261, z=10236, k=274 digits : 0065 ... 6631
n=71, x=1847, y=5680, z=64327, k=330 digits : 0036 ... 0301

7.22 n=73

7.22.1 n=73, x=2 to 19999

n=73, x=2, y=293, z=439, k=188 digits : 3087 ... 7171
n=73, x=2, y=877, z=1317, k=222 digits : 0023 ... 3196
n=73, x=3, y=877, z=2344, k=240 digits : 1647 ... 7446
n=73, x=3, y=877, z=5860, k=268 digits : 7387 ... 0716
n=73, x=3, y=1607, z=2930, k=246 digits : 0085 ... 6279
n=73, x=3, y=4829, z=28064, k=317 digits : 0001 ... 0099
n=73, x=3, y=17287, z=47026, k=332 digits : 4937 ... 4871
n=73, x=4, y=293, z=1317, k=222 digits : 0034 ... 6720
n=73, x=4, y=439, z=1607, k=228 digits : 3878 ... 5572
n=73, x=4, y=2051, z=2195, k=237 digits : 0004 ... 9252
n=73, x=4, y=4385, z=38969, k=327 digits : 0193 ... 3782
n=73, x=4, y=4673, z=33729, k=322 digits : 0055 ... 2944
n=73, x=5, y=3943, z=36813, k=325 digits : 0002 ... 0023
n=73, x=7, y=877, z=5707, k=267 digits : 0471 ... 5471
n=73, x=7, y=19283, z=175215, k=373 digits : 0002 ... 0287
n=73, x=8, y=439, z=879, k=209 digits : 0002 ... 6759
n=73, x=9, y=293, z=878, k=209 digits : 0003 ... 7691
n=73, x=9, y=1172, z=9929, k=284 digits : 5675 ... 4624
n=73, x=9, y=9083, z=9929, k=283 digits : 0731 ... 9919
n=73, x=9, y=15188, z=146237, k=367 digits : 0560 ... 4285

n=73, x=12, y=11401, z=20149, k=305 digits : 0005 ... 6677
n=73, x=14, y=2637, z=19283, k=304 digits : 9232 ... 9763
n=73, x=16, y=6139, z=18251, k=302 digits : 0066 ... 1589
n=73, x=17, y=879, z=6863, k=273 digits : 0001 ... 7503
n=73, x=18, y=1465, z=1753, k=230 digits : 0013 ... 5408
n=73, x=27, y=14747, z=84974, k=350 digits : 0020 ... 1355
n=73, x=31, y=4385, z=9376, k=281 digits : 0007 ... 7506
n=73, x=64, y=293, z=1317, k=221 digits : 0002 ... 0000
n=73, x=79, y=8765, z=88239, k=351 digits : 0176 ... 5087
n=73, x=105, y=586, z=6571, k=271 digits : 0120 ... 2911
n=73, x=109, y=1317, z=1753, k=229 digits : 0002 ... 9423
n=73, x=130, y=3951, z=12271, k=289 digits : 0004 ... 4672
n=73, x=204, y=5567, z=44531, k=329 digits : 0004 ... 9555
n=73, x=209, y=6585, z=9929, k=282 digits : 0043 ... 1311
n=73, x=302, y=1753, z=6585, k=270 digits : 0016 ... 6416
n=73, x=438, y=439, z=3943, k=254 digits : 0041 ... 9096
n=73, x=439, y=1205, z=5259, k=263 digits : 0151 ... 8767
n=73, x=439, y=2190, z=20149, k=304 digits : 8381 ... 7507
n=73, x=439, y=9945, z=89944, k=351 digits : 0111 ... 6182
n=73, x=511, y=1753, z=9929, k=282 digits : 0066 ... 5855
n=73, x=529, y=1465, z=9929, k=282 digits : 0077 ... 3055
n=73, x=584, y=8761, z=43801, k=328 digits : 3005 ... 0284
n=73, x=859, y=3512, z=11243, k=286 digits : 0015 ... 6718
n=73, x=1465, y=1756, z=8761, k=278 digits : 0028 ... 1936
n=73, x=1607, y=15987, z=128627, k=361 digits : 0002 ... 5191
n=73, x=1753, y=5987, z=13185, k=290 digits : 0042 ... 0783
n=73, x=1754, y=13771, z=19755, k=302 digits : 0080 ... 9040
n=73, x=1756, y=11829, z=65857, k=340 digits : 4186 ... 3289
n=73, x=2051, y=2631, z=9491, k=280 digits : 4308 ... 4599
n=73, x=3223, y=3506, z=10513, k=283 digits : 0324 ... 7091
n=73, x=3223, y=19273, z=129944, k=361 digits : 0002 ... 6868
n=73, x=3943, y=9201, z=18166, k=300 digits : 1279 ... 2004
n=73, x=4673, y=10658, z=95923, k=352 digits : 1002 ... 5891
n=73, x=5567, y=6146, z=29801, k=315 digits : 0407 ... 4579
n=73, x=6139, y=13771, z=79459, k=345 digits : 0007 ... 4119
n=73, x=9685, y=11243, z=40298, k=324 digits : 3494 ... 1456
n=73, x=10255, y=13436, z=65411, k=339 digits : 0386 ... 5335
n=73, x=11243, y=16661, z=161192, k=367 digits : 0453 ... 7168

7.23 n=79

7.23.1 n=79, x=2 to 19999

n=79, x=4, y=11379, z=45979, k=360 digits : 1050 ... 5254
n=79, x=5, y=8059, z=79884, k=378 digits : 0061 ... 9360
n=79, x=6, y=8849, z=18329, k=328 digits : 6306 ... 5524
n=79, x=11, y=2219, z=2846, k=266 digits : 0011 ... 2699
n=79, x=11, y=17752, z=132339, k=395 digits : 0158 ... 3785

n=79, x=15, y=6638, z=49613, k=362 digits : 0018 ... 2735
n=79, x=15, y=16433, z=162938, k=402 digits : 0013 ... 0048
n=79, x=21, y=2536, z=18013, k=328 digits : 1619 ... 0835
n=79, x=26, y=4755, z=7901, k=299 digits : 0845 ... 3175
n=79, x=37, y=2536, z=7901, k=300 digits : 1113 ... 8541
n=79, x=69, y=12641, z=83297, k=378 digits : 0073 ... 1511
n=79, x=86, y=18803, z=66361, k=370 digits : 0079 ... 6991
n=79, x=91, y=10429, z=62572, k=369 digits : 0001 ... 4766
n=79, x=92, y=2219, z=5531, k=287 digits : 0425 ... 4428
n=79, x=97, y=15172, z=16433, k=323 digits : 0454 ... 0348
n=79, x=128, y=2853, z=15653, k=322 digits : 0041 ... 2604
n=79, x=129, y=18703, z=102553, k=385 digits : 0002 ... 5871
n=79, x=158, y=1423, z=2687, k=263 digits : 0135 ... 3208
n=79, x=181, y=5531, z=13314, k=316 digits : 4961 ... 2648
n=79, x=232, y=15653, z=92445, k=381 digits : 0006 ... 2517
n=79, x=249, y=10903, z=51988, k=362 digits : 0025 ... 8476
n=79, x=284, y=5689, z=57377, k=365 digits : 0009 ... 4570
n=79, x=317, y=1423, z=16593, k=324 digits : 3161 ... 8959
n=79, x=317, y=5515, z=55467, k=364 digits : 6195 ... 7615
n=79, x=331, y=2853, z=6163, k=290 digits : 0042 ... 6671
n=79, x=741, y=6340, z=66361, k=370 digits : 0027 ... 6947
n=79, x=855, y=17698, z=32233, k=345 digits : 0002 ... 6903
n=79, x=932, y=951, z=10271, k=307 digits : 0908 ... 8246
n=79, x=1423, y=6241, z=18013, k=325 digits : 0009 ... 6231
n=79, x=1437, y=13435, z=43612, k=355 digits : 0401 ... 8070
n=79, x=1585, y=2687, z=5692, k=287 digits : 0191 ... 1132
n=79, x=1711, y=8849, z=37936, k=350 digits : 0096 ... 6244
n=79, x=1775, y=16593, z=92918, k=381 digits : 0001 ... 6272
n=79, x=2208, y=12641, z=83297, k=377 digits : 0002 ... 3600
n=79, x=2213, y=6657, z=45830, k=357 digits : 0002 ... 6151
n=79, x=2465, y=6163, z=30813, k=343 digits : 0870 ... 4015
n=79, x=3811, y=18809, z=49297, k=359 digits : 0152 ... 3351
n=79, x=4181, y=11379, z=65732, k=369 digits : 0001 ... 5246
n=79, x=4917, y=9961, z=49297, k=359 digits : 0223 ... 9351

no more soluytions to 19999 TBC

7.24 n=83

7.24.1 n=83, x=2 to 19999

n=83, x=2, y=499, z=997, k=243 digits : 0783 ... 4311
n=83, x=2, y=835, z=997, k=243 digits : 0468 ... 9951
n=83, x=2, y=1503, z=2495, k=276 digits : 1207 ... 8192
n=83, x=2, y=7971, z=8483, k=318 digits : 0086 ... 3088
n=83, x=3, y=997, z=5314, k=303 digits : 0102 ... 8956
n=83, x=3, y=998, z=2171, k=271 digits : 0134 ... 5308
n=83, x=3, y=7972, z=54835, k=385 digits : 0001 ... 3780

n=83, x=4, y=501, z=997, k=243 digits : 0390 ... 6516
n=83, x=4, y=2171, z=5815, k=305 digits : 0005 ... 3085
n=83, x=4, y=4175, z=6979, k=311 digits : 0929 ... 8071
n=83, x=4, y=4649, z=16533, k=342 digits : 0043 ... 2993
n=83, x=5, y=997, z=2657, k=278 digits : 0012 ... 0031
n=83, x=5, y=6979, z=12024, k=331 digits : 0105 ... 2197
n=83, x=6, y=9965, z=34541, k=368 digits : 2327 ... 0192
n=83, x=7, y=9629, z=12191, k=331 digits : 0168 ... 1143
n=83, x=8, y=499, z=2171, k=271 digits : 0101 ... 5875
n=83, x=8, y=9629, z=69973, k=393 digits : 0002 ... 8581
n=83, x=9, y=9304, z=69361, k=393 digits : 0001 ... 8453
n=83, x=12, y=1163, z=2495, k=275 digits : 0260 ... 3245
n=83, x=12, y=1163, z=4175, k=293 digits : 0005 ... 9553
n=83, x=13, y=1837, z=1993, k=267 digits : 0151 ... 8879
n=83, x=13, y=2657, z=3489, k=286 digits : 0091 ... 9671
n=83, x=20, y=6179, z=12119, k=330 digits : 0056 ... 9851
n=83, x=21, y=499, z=835, k=236 digits : 3613 ... 5871
n=83, x=31, y=501, z=997, k=242 digits : 0050 ... 8983
n=83, x=31, y=2495, z=2991, k=281 digits : 0001 ... 6239
n=83, x=35, y=3988, z=24883, k=356 digits : 2085 ... 9676
n=83, x=47, y=499, z=3507, k=287 digits : 0206 ... 4551
n=83, x=49, y=1670, z=15439, k=339 digits : 0357 ... 2811
n=83, x=52, y=1163, z=4491, k=295 digits : 0513 ... 0976
n=83, x=52, y=2657, z=3489, k=286 digits : 0022 ... 1408
n=83, x=56, y=499, z=835, k=236 digits : 1355 ... 8514
n=83, x=70, y=997, z=1837, k=263 digits : 0650 ... 8196
n=83, x=83, y=167, z=499, k=218 digits : 0012 ... 7991
n=83, x=83, y=997, z=1993, k=266 digits : 0043 ... 4279
n=83, x=93, y=499, z=1336, k=252 digits : 4462 ... 3250
n=83, x=106, y=7181, z=48309, k=379 digits : 0161 ... 2428
n=83, x=111, y=997, z=1996, k=266 digits : 0037 ... 0501
n=83, x=119, y=2994, z=10967, k=326 digits : 0054 ... 9760
n=83, x=125, y=1994, z=12119, k=330 digits : 0028 ... 0423
n=83, x=129, y=1670, z=12119, k=330 digits : 0032 ... 5751
n=83, x=166, y=499, z=1163, k=247 digits : 0287 ... 7756
n=83, x=167, y=203, z=1994, k=267 digits : 0111 ... 0251
n=83, x=167, y=249, z=1163, k=247 digits : 0573 ... 8215
n=83, x=167, y=388, z=2495, k=274 digits : 0056 ... 8162
n=83, x=167, y=425, z=3992, k=291 digits : 0279 ... 0922
n=83, x=167, y=1328, z=12119, k=330 digits : 0031 ... 6251
n=83, x=167, y=1497, z=4652, k=296 digits : 2231 ... 4064
n=83, x=167, y=1951, z=7971, k=315 digits : 0257 ... 2271
n=83, x=167, y=2905, z=17597, k=343 digits : 0275 ... 7223
n=83, x=167, y=3984, z=12119, k=330 digits : 0010 ... 5481
n=83, x=208, y=2657, z=3489, k=285 digits : 0005 ... 7596
n=83, x=218, y=1497, z=4985, k=298 digits : 0049 ... 0752
n=83, x=290, y=1497, z=2657, k=276 digits : 1453 ... 3532
n=83, x=446, y=835, z=7971, k=315 digits : 0225 ... 1120

n=83, x=467, y=499, z=7971, k=315 digits : 0360 ... 7743
n=83, x=499, y=668, z=9463, k=321 digits : 0003 ... 8901
n=83, x=499, y=1868, z=7971, k=314 digits : 0090 ... 2940
n=83, x=499, y=1994, z=10467, k=324 digits : 4242 ... 0440
n=83, x=499, y=2505, z=10459, k=324 digits : 3171 ... 5511
n=83, x=499, y=9965, z=58269, k=385 digits : 0001 ... 9199
n=83, x=501, y=539, z=2657, k=276 digits : 2337 ... 1751
n=83, x=768, y=997, z=13285, k=333 digits : 0001 ... 8432
n=83, x=862, y=1163, z=14955, k=337 digits : 0002 ... 8640
n=83, x=903, y=3992, z=12119, k=329 digits : 0001 ... 6426
n=83, x=996, y=5479, z=19423, k=345 digits : 0008 ... 0106
n=83, x=997, y=1028, z=17445, k=342 digits : 0064 ... 0160
n=83, x=997, y=1163, z=13360, k=333 digits : 0001 ... 8823
n=83, x=997, y=1163, z=14970, k=337 digits : 0002 ... 2824
n=83, x=997, y=1163, z=7984, k=314 digits : 0082 ... 0791
n=83, x=997, y=5979, z=21256, k=349 digits : 0001 ... 8568
n=83, x=1125, y=1994, z=12119, k=329 digits : 0003 ... 7547
n=83, x=1163, y=1702, z=7971, k=314 digits : 0042 ... 1036
n=83, x=1163, y=8016, z=68363, k=390 digits : 0030 ... 7976
n=83, x=2171, y=3493, z=9304, k=319 digits : 0355 ... 3508
n=83, x=2495, y=2517, z=17597, k=342 digits : 0021 ... 3447
n=83, x=4649, y=4990, z=12119, k=328 digits : 3011 ... 6999
n=83, x=4652, y=9965, z=51937, k=380 digits : 1006 ... 9717
n=83, x=5815, y=9629, z=19539, k=345 digits : 0001 ... 8287
n=83, x=5815, y=9686, z=51461, k=379 digits : 0389 ... 0555
n=83, x=6143, y=6979, z=81162, k=395 digits : 0860 ... 4163
n=83, x=7971, y=9908, z=166499, k=421 digits : 0001 ... 8368
n=83, x=7976, y=8373, z=83333, k=396 digits : 4812 ... 4566
n=83, x=8467, y=8973, z=93040, k=400 digits : 3550 ... 4538
n=83, x=10956, y=17597, z=83333, k=396 digits : 1666 ... 1858
n=83, x=15469, y=18596, z=92965, k=399 digits : 0877 ... 6378

7.25 n=89

7.25.1 n=89, x=2 to 19999

n=89, x=2, y=11751, z=98279, k=435 digits : 0923 ... 5632
n=89, x=2, y=16557, z=142757, k=450 digits : 0012 ... 8556
n=89, x=3, y=8546, z=18437, k=371 digits : 0937 ... 3423
n=89, x=3, y=10561, z=81187, k=428 digits : 3419 ... 1943
n=89, x=3, y=11393, z=64799, k=419 digits : 0766 ... 7463
n=89, x=4, y=7121, z=13425, k=359 digits : 0633 ... 0920
n=89, x=4, y=9621, z=49225, k=409 digits : 0002 ... 9385
n=89, x=4, y=11751, z=14419, k=362 digits : 0020 ... 4359
n=89, x=6, y=14141, z=27419, k=386 digits : 0041 ... 0603
n=89, x=8, y=1969, z=11217, k=353 digits : 0001 ... 9860
n=89, x=8, y=8011, z=14499, k=362 digits : 0024 ... 3715
n=89, x=8, y=17623, z=116887, k=441 digits : 0006 ... 2792

n=89, x=11, y=3580, z=20311, k=375 digits : 0305 ... 0265
n=89, x=13, y=5345, z=8413, k=341 digits : 0003 ... 5095
n=89, x=13, y=12888, z=103693, k=437 digits : 0001 ... 8016
n=89, x=14, y=1969, z=2137, k=289 digits : 0003 ... 6892
n=89, x=16, y=1069, z=5549, k=326 digits : 0018 ... 5304
n=89, x=16, y=1969, z=19585, k=374 digits : 0015 ... 6389
n=89, x=16, y=12709, z=97925, k=434 digits : 0077 ... 7998
n=89, x=17, y=5519, z=8902, k=343 digits : 0382 ... 8016
n=89, x=20, y=1069, z=1969, k=286 digits : 0036 ... 2905
n=89, x=32, y=1611, z=10859, k=351 digits : 0273 ... 1537
n=89, x=37, y=3759, z=13897, k=360 digits : 2715 ... 2151
n=89, x=37, y=4451, z=11814, k=354 digits : 0014 ... 1292
n=89, x=50, y=3401, z=31001, k=391 digits : 0102 ... 6728
n=89, x=89, y=179, z=2137, k=289 digits : 0006 ... 5167
n=89, x=101, y=1069, z=2685, k=297 digits : 0005 ... 4343
n=89, x=101, y=7483, z=22431, k=377 digits : 0009 ... 5479
n=89, x=141, y=7477, z=27781, k=386 digits : 0010 ... 4519
n=89, x=147, y=1069, z=6802, k=333 digits : 0001 ... 4256
n=89, x=199, y=10859, z=14242, k=360 digits : 1512 ... 1747
n=89, x=323, y=2864, z=18691, k=370 digits : 0086 ... 4532
n=89, x=452, y=9613, z=21365, k=375 digits : 0237 ... 7066
n=89, x=657, y=716, z=9257, k=344 digits : 2381 ... 2546
n=89, x=716, y=8013, z=9257, k=343 digits : 0195 ... 0033
n=89, x=3739, y=5907, z=14959, k=361 digits : 0001 ... 6239
n=89, x=4117, y=4451, z=36329, k=395 digits : 0109 ... 0247
n=89, x=5143, y=11927, z=155931, k=450 digits : 0015 ... 6951

7.26 n=97

7.26.1 n=97, x=2 to 19999

n=97, x=2, y=1747, z=4659, k=349 digits : 0004 ... 7760
n=97, x=3, y=778, z=5821, k=359 digits : 0117 ... 9935
n=97, x=4, y=971, z=1167, k=291 digits : 0707 ... 0109
n=97, x=7, y=13196, z=63407, k=457 digits : 0001 ... 9646
n=97, x=7, y=17463, z=112327, k=480 digits : 5744 ... 0751
n=97, x=9, y=10871, z=11447, k=385 digits : 0004 ... 2319
n=97, x=13, y=9725, z=41913, k=439 digits : 0440 ... 5247
n=97, x=16, y=1945, z=3881, k=341 digits : 0001 ... 2089
n=97, x=19, y=971, z=5835, k=358 digits : 0018 ... 7143
n=97, x=19, y=11643, z=16507, k=400 digits : 3558 ... 1175
n=97, x=19, y=15536, z=71187, k=461 digits : 0002 ... 9904
n=97, x=23, y=15530, z=27743, k=421 digits : 0009 ... 0212
n=97, x=28, y=971, z=1167, k=291 digits : 0101 ... 2763
n=97, x=33, y=1747, z=1945, k=311 digits : 0945 ... 5087
n=97, x=37, y=8735, z=16727, k=400 digits : 8682 ... 9887
n=97, x=45, y=971, z=1556, k=302 digits : 0062 ... 4086
n=97, x=45, y=3299, z=6224, k=360 digits : 1145 ... 4805

n=97, x=52, y=8947, z=42487, k=439 digits : 0441 ... 0561
n=97, x=85, y=1553, z=2913, k=328 digits : 2858 ... 5183
n=97, x=103, y=8537, z=42012, k=438 digits : 0079 ... 3616
n=97, x=109, y=2913, z=4657, k=347 digits : 0432 ... 9295
n=97, x=114, y=3881, z=33065, k=429 digits : 0001 ... 5136
n=97, x=121, y=1945, z=3881, k=340 digits : 1468 ... 8271
n=97, x=167, y=2723, z=8735, k=373 digits : 0005 ... 2607
n=97, x=169, y=13971, z=57235, k=451 digits : 0230 ... 9303
n=97, x=194, y=1553, z=4657, k=347 digits : 0456 ... 8320
n=97, x=209, y=1167, z=3884, k=340 digits : 1526 ... 9224
n=97, x=287, y=16103, z=144679, k=489 digits : 0005 ... 1391
n=97, x=304, y=971, z=5835, k=357 digits : 0001 ... 8480
n=97, x=305, y=778, z=2913, k=328 digits : 1590 ... 5816
n=97, x=340, y=1553, z=2913, k=327 digits : 0714 ... 6452
n=97, x=376, y=11447, z=17463, k=401 digits : 0004 ... 3920
n=97, x=389, y=461, z=7765, k=369 digits : 0001 ... 7655
n=97, x=389, y=971, z=12424, k=388 digits : 2964 ... 9434
n=97, x=389, y=1391, z=8735, k=373 digits : 0004 ... 9631
n=97, x=389, y=13196, z=18449, k=403 digits : 0665 ... 0109
n=97, x=412, y=4855, z=11447, k=384 digits : 2154 ... 0096
n=97, x=667, y=971, z=11643, k=385 digits : 0003 ... 9575
n=97, x=788, y=4855, z=17463, k=401 digits : 0004 ... 6080
n=97, x=849, y=5821, z=19405, k=405 digits : 0008 ... 3303
n=97, x=922, y=971, z=4659, k=347 digits : 0160 ... 0452
n=97, x=971, y=2619, z=11447, k=384 digits : 1694 ... 7119
n=97, x=1528, y=4657, z=29105, k=422 digits : 0048 ... 8256
n=97, x=1553, y=1722, z=29105, k=423 digits : 0129 ... 0240
n=97, x=1553, y=7566, z=31817, k=426 digits : 0015 ... 7988
n=97, x=1553, y=8197, z=34341, k=429 digits : 0002 ... 8951
n=97, x=1637, y=2913, z=29105, k=422 digits : 0072 ... 9055
n=97, x=1661, y=7765, z=34341, k=429 digits : 0002 ... 2039
n=97, x=1697, y=4659, z=11447, k=383 digits : 0545 ... 6951
n=97, x=1746, y=3881, z=31817, k=426 digits : 0026 ... 7200
n=97, x=1784, y=14551, z=69855, k=458 digits : 0042 ... 2439
n=97, x=1867, y=1945, z=31817, k=426 digits : 0049 ... 1455
n=97, x=2704, y=13971, z=57235, k=450 digits : 0014 ... 1760
n=97, x=2913, y=7226, z=31817, k=425 digits : 0008 ... 7508
n=97, x=2923, y=11447, z=58215, k=450 digits : 0082 ... 3119
n=97, x=3286, y=4659, z=57235, k=450 digits : 0035 ... 7664
n=97, x=3299, y=17470, z=117089, k=479 digits : 0655 ... 2459
n=97, x=4463, y=18672, z=27167, k=418 digits : 0055 ... 6261

7.27 n=101

7.27.1 n=101, x=2 to 19999

n=97, x=2, y=1747, z=4659, k=349 digits : 0004 ... 7760
n=97, x=3, y=778, z=5821, k=359 digits : 0117 ... 9935

n=97, x=4, y=971, z=1167, k=291 digits : 0707 ... 0109
n=97, x=7, y=13196, z=63407, k=457 digits : 0001 ... 9646
n=97, x=7, y=17463, z=112327, k=480 digits : 5744 ... 0751
n=97, x=9, y=10871, z=11447, k=385 digits : 0004 ... 2319
n=97, x=13, y=9725, z=41913, k=439 digits : 0440 ... 5247
n=97, x=16, y=1945, z=3881, k=341 digits : 0001 ... 2089
n=97, x=19, y=971, z=5835, k=358 digits : 0018 ... 7143
n=97, x=19, y=11643, z=16507, k=400 digits : 3558 ... 1175
n=97, x=19, y=15536, z=71187, k=461 digits : 0002 ... 9904
n=97, x=23, y=15530, z=27743, k=421 digits : 0009 ... 0212
n=97, x=28, y=971, z=1167, k=291 digits : 0101 ... 2763
n=97, x=33, y=1747, z=1945, k=311 digits : 0945 ... 5087
n=97, x=37, y=8735, z=16727, k=400 digits : 8682 ... 9887
n=97, x=45, y=971, z=1556, k=302 digits : 0062 ... 4086
n=97, x=45, y=3299, z=6224, k=360 digits : 1145 ... 4805
n=97, x=52, y=8947, z=42487, k=439 digits : 0441 ... 0561
n=97, x=85, y=1553, z=2913, k=328 digits : 2858 ... 5183
n=97, x=103, y=8537, z=42012, k=438 digits : 0079 ... 3616
n=97, x=109, y=2913, z=4657, k=347 digits : 0432 ... 9295
n=97, x=114, y=3881, z=33065, k=429 digits : 0001 ... 5136
n=97, x=121, y=1945, z=3881, k=340 digits : 1468 ... 8271
n=97, x=167, y=2723, z=8735, k=373 digits : 0005 ... 2607
n=97, x=169, y=13971, z=57235, k=451 digits : 0230 ... 9303
n=97, x=194, y=1553, z=4657, k=347 digits : 0456 ... 8320
n=97, x=209, y=1167, z=3884, k=340 digits : 1526 ... 9224
n=97, x=287, y=16103, z=144679, k=489 digits : 0005 ... 1391
n=97, x=304, y=971, z=5835, k=357 digits : 0001 ... 8480
n=97, x=305, y=778, z=2913, k=328 digits : 1590 ... 5816
n=97, x=340, y=1553, z=2913, k=327 digits : 0714 ... 6452
n=97, x=376, y=11447, z=17463, k=401 digits : 0004 ... 3920
n=97, x=389, y=461, z=7765, k=369 digits : 0001 ... 7655
n=97, x=389, y=971, z=12424, k=388 digits : 2964 ... 9434
n=97, x=389, y=1391, z=8735, k=373 digits : 0004 ... 9631
n=97, x=389, y=13196, z=18449, k=403 digits : 0665 ... 0109
n=97, x=412, y=4855, z=11447, k=384 digits : 2154 ... 0096
n=97, x=667, y=971, z=11643, k=385 digits : 0003 ... 9575
n=97, x=788, y=4855, z=17463, k=401 digits : 0004 ... 6080
n=97, x=849, y=5821, z=19405, k=405 digits : 0008 ... 3303
n=97, x=922, y=971, z=4659, k=347 digits : 0160 ... 0452
n=97, x=971, y=2619, z=11447, k=384 digits : 1694 ... 7119
n=97, x=1528, y=4657, z=29105, k=422 digits : 0048 ... 8256
n=97, x=1553, y=1722, z=29105, k=423 digits : 0129 ... 0240
n=97, x=1553, y=7566, z=31817, k=426 digits : 0015 ... 7988
n=97, x=1553, y=8197, z=34341, k=429 digits : 0002 ... 8951
n=97, x=1637, y=2913, z=29105, k=422 digits : 0072 ... 9055
n=97, x=1661, y=7765, z=34341, k=429 digits : 0002 ... 2039
n=97, x=1697, y=4659, z=11447, k=383 digits : 0545 ... 6951
n=97, x=1746, y=3881, z=31817, k=426 digits : 0026 ... 7200

n=97, x=1784, y=14551, z=69855, k=458 digits : 0042 ... 2439
n=97, x=1867, y=1945, z=31817, k=426 digits : 0049 ... 1455
n=97, x=2704, y=13971, z=57235, k=450 digits : 0014 ... 1760
n=97, x=2913, y=7226, z=31817, k=425 digits : 0008 ... 7508
n=97, x=2923, y=11447, z=58215, k=450 digits : 0082 ... 3119
n=97, x=3286, y=4659, z=57235, k=450 digits : 0035 ... 7664
n=97, x=3299, y=17470, z=117089, k=479 digits : 0655 ... 2459
n=97, x=4463, y=18672, z=27167, k=418 digits : 0055 ... 6261

8 Special Solutions

Special solutions are generally those that lead to a low value for k.

They are characterised by having x and/or y divisible by the exponent n or factors of the form $2mn+1$. The latter $2mn+1$ condition leads to non-trivial unity roots (4.2), which, in turn, means that a relatively low k-value may be obtained.

Note too that z may also be divisible by n or factors of the $2mn+1$ form.

8.1 Non-trivial Unity Roots

This section lists complete solutions (2.16), for which the unity roots are all or mostly non-trivial (4.2).

Each solution is split into three separate tables, giving all the variables comprising the 'complete set' (2.17).

8.1.1 n=3, x=9 to 39, 91

In none of the cases is z less than the sum of x and y, i.e. $z < x + y$, which is a necessity to get any small k-values, see [3].

All x have factors of n or $2mn+1$, for $n = 3$, which means non-trivial roots are possible, i.e. P, Q, R and their conjugates $\overline{P}, \overline{Q}, \overline{R}$ are not ± 1, and hence low k-values.

n	x	y	z	k
3	9	31	70	16
3	13	14	61	20
3	14	19	97	35
3	19	21	52	6 Note **
3	39	76	619	129
3	91	185	516	15
3	133	632	1005	9
3	151	279	910	19
3	3708	4355	15325	14

Note *. The semi-analytic determination of (9,31,70) is fully documented in [1]#4 and repeated above in Section (3), but the unity roots are not the same – remember they are non-unique – see also Section (2).

Note **. The (19,21,52) solution has an exceptionally small k-value of 6 and is currently the record holder for the smallest k-value, for all odd, prime exponents, $n \geq 3$, where $z > y$ as per condition (1.1a).

n	x, y, z	P	Q	R	$\overline{P}$	$\overline{Q}$	$\overline{R}$
3	9,31,70	7	25	-11	-5	5	-51
3	13,14,61	3	9	-47	-4	11	-13
3	14,19,97	9	11	-35	-3	7	-61
3	19,21,52	11	16	-9	-12	4	-29
3	39,76,619	16	49	-366	-17	45	-252
3	91,185,516	9	26	-337	-10	121	-49
3	133,632,1005	102	-135	-841	30	529	-766
3	151,279,910	32	-212	-81	118	25	-191
3	3708,4355,15325	3445	-2844	-3873	2425	1101	-14235

n	x, y, z	α	β	γ	K	V
3	9,31,70	4	-4	-8	651	-650
3	13,14,61	1	-7	-10	698	-697
3	14,19,97	2	-4	-22	2185	-2184

3	19,21,52	7	-3	-5	193	-192
3	39,76,619	7	-29	-149	94165	-94164
3	91,185,516	1	-17	-32	19569	-19568
3	133,632,1005	-23	113	-641	575851	-575850
3	151,279,910	-25	19	-17	13947	-13946
3	3708,4355,15325	-2253	719	-3598	60355036	-60355035

All x, y, z have factors of n or $2mn+1$, for $n=3$, deconstructed as follows, in the order they appear in the above tables. $\overline{P}$, $\overline{Q}$

For the x values

9=3^2,
13=4.3+1 (m=2),
14=2.7, where 7=2.3+1 (m=1)
19=6.3+1 (m=3).
39=13.3, see above for 13
91=13.7, see above for 13,7
133=44.3+1
151=50.3+1
3708=36.103, 103=34.3+1

For the y values

31=10.3+1 (m=5)
14=2.7, where 7=2.3+1 (m=1)
19=6.3+1 (m=3)
21=3.7, i.e. divisible by both n and $2mn+1$
76=4.19, see x for 19
185= 5.37, 37=12.3 + 1
632=8.79, 79=26.3+1
279=9.31, 31=10.3+1
4355=5.871, 871=290.3+1

For the z values

70=10.7, were 7=2.3+1 (m=1)
61 = 20.3+1, (m=10)
97=32.3+1, (m=16)
52=4.13, see x for 13
619=206.3+1, (m=103)
516=12.43, 43=14.3+1
1005=15.67=22.3+1
910=10.91, 91=30.3+1
15325=25.613, 613=204.3+1

8.1.2 n=5, x=5

This is the quintic solution (5,11,31) first presented in [1]#1, Appendix(C2). It is the only quintic solution currently known with mostly non-trivial unity roots – albeit P and $\overline{P}$ are trivially unity. There may be other quintic solutions already given in Section (7.3.19) that may also have non-trivial roots, but the unity roots have not yet been determined for these reduced solutions.

n	x	y	z	k
5	5	11	31	16695

n	x,y,z	P	Q	R	$\overline{P}$	$\overline{Q}$	$\overline{R}$
5	5,11,31	1	4	-8	1	3	-4

n	x,y,z	α	β	γ	K	V
5	5,11,31	0	-1	-1	45	-44

9 Miscellaneous Solutions

9.1 Arbitrary z, not limited to greater than x or y

This section lists solutions where z is not necessarily greater than x or y.

9.1.1 n=3, x=7 to 49

All x have factors of n or $2mn+1$ for $n=3$.

The unity roots P and Q are non-trivial, i.e. not equal to unity (4.2), but note that R and or $\overline{R}$ are commonly -1, especially if z is less than y (or x).

The tables are split into two, the first table has the solution (x, y, z) and all the full unity roots, whilst the following table also shows the same solution (x, y, z) but also gives the dual solution (α, β, γ), (2.11), the all-important k-value, and the Kinetic (K) and Potential (V) energy terms (2.6), where the reader can verify the DCE (2.8) is satisfied for unity eigenvalue.

n	**x**	**y**	**z**	**P**	**Q**	**R**	$\overline{P}$	$\overline{Q}$	$\overline{R}$	
3	7	9	4	4	7	-1	-5	4	-1	
3	9	19	4	7	11	-1	-5	7	-1	
3	9	31	70	7	25	-11	-5	5	-51	Note *
3	13	14	3	9	11	-1	-10	9	-1	
3	13	14	61	3	9	-47	-4	11	-13	Note *
3	13	35	3	3	11	-1	-4	16	2	
3	14	19	3	11	7	-1	-5	11	-1	
3	14	19	97	9	11	-35	-3	7	-61	Note *
3	19	21	52	11	16	-9	-12	4	-29	Note *
3	39	76	7	22	45	-4	-23	49	-2	
3	39	76	619	16	49	-366	-17	45	-252	Note *
3	49	74	9	30	47	-7	-31	63	-4	

n	**x**	**y**	**z**	α	β	γ	**k**	**K**	**V**	
3	7	9	4	3	-3	0	-4	9	-8	
3	9	19	4	4	-4	0	-11	43	-42	
3	9	31	70	4	-4	-8	16	651	-650	Note *
3	13	14	3	7	-7	0	-9	10	-9	
3	13	14	61	1	-7	-10	20	698	-697	Note *
3	13	35	3	1	-5	1	-33	162	-161	
3	14	19	3	4	-4	0	-12	23	-22	
3	14	19	97	2	-4	-22	35	2185	-2184	Note *
3	19	21	52	7	-3	-5	6	193	-192	Note *
3	39	76	7	13	-29	-1	-24	1707	-1706	
3	39	76	619	7	-29	-149	129	94165	-94164	Note *
3	49	74	9	19	-40	-3	-16	2059	-2058	

Note *. These are repeats of those given in Section (8.1.1)

From the second table, it can be seen that all k-values are less than 100, excepting the solution (39,76,619).

9.2 Even exponent n=4

9.2.1 n=4, x=2 to 37

The complete solutions for the even exponent, $n = 4$, are tabulated below, each solution split across three consecutive tables.

The solutions are selected such that they meet conditions (1.1).

Note that some have trivial unity roots (4.1).

They have only been given up to x=37.

n	x	y	z	k
4	3	10	17	144
4	3	40	41	54
4	4	15	17	32
4	4	15	41	1128
4	5	11	17	73
4	5	29	82	3743
4	10	17	41	392
4	12	37	113	3212
4	15	16	17	-8
4	16	19	41	211
4	17	33	137	4567
4	20	21	113	3428
4	25	39	73	361
4	25	32	193	8977
4	26	41	193	6728
4	27	40	41	-6 Note *
4	35	48	113	823
4	37	68	73	28

n	x,y,z	P	Q	R	$\overline{P}$	$\overline{Q}$	$\overline{R}$
4	3,10,17	-1	9	-15	-1	9	9
4	3,40,41	-1	27	-3	-1	3	27
4	4,15,17	-1	8	-2	-1	2	8
4	4,15,41	-1	14	-38	-1	14	14
4	5,11,17	3	-23	2	12	-1	-8
4	5,29,82	-3	28	-79	-2	28	55
4	10,17,41	-3	16	-38	-7	16	14
4	12,37,113	-7	31	-18	-7	6	69

4	15,16,17	8	-1	2	2	-1	-8
4	16,19,41	-5	18	-38	-13	18	14
4	17,33,137	-1	10	-41	-1	10	10
4	20,21,133	-3	13	-69	-7	13	18
4	25,39,73	-7	31	-63	-18	34	22
4	25,32,193	-1	9	-150	-1	25	9
4	26,41,193	-1	9	-150	-1	32	9
4	27,40,41	-1	3	-27	-1	27	3
4	35,48,113	-29	43	-44	-29	19	95
4	37,68,73	-31	13	-22	-6	21	63

n	x,y,z	α	β	γ	K	V
4	3,10,17	0	-8	8	-53	54
4	3,40,41	0	-2	2	1	0 Note **
4	4,15,17	0	-1	1	1	0 Note **
4	4,15,41	0	-13	13	-335	336
4	5,11,17	-7	-2	1	43	-42 Note ***
4	5,29,82	-1	-27	53	-3555	3556
4	10,17,41	-2	-15	13	-255	256
4	12,37,113	-4	-5	11	-1007	1008
4	15,16,17	-1	0	1	1	0 Note **
4	16,19,41	-4	-17	13	-143	144
4	17,33,137	0	-3	-3	-309	310
4	20,21,133	-1	-8	11	-1052	1053
4	25,39,73	-5	-27	19	-206	207
4	25,32,193	0	-7	-7	-1124	1125
4	26,41,193	0	-7	-7	-1061	1062
4	27,40,41	0	-2	2	1	0 Note **
4	35,48,113	-24	-17	37	-2522	2523
4	37,68,73	-5	-4	19	-927	928

Note *. This solution has a very low absolute k-value of 6. This actually matches the record for the cubic solution (19,21,52) below, Section (10.1) but, as it is an even-exponent, the solution is not given such importance. Much the same remarks apply to the (15,16,17) solution above it, which has a low, absolute k-value of 8. See also the next comment about this solution.

Note **. These solutions (3,40,41), (4,15,17), (15,16,17) and (19,21,52) are rare outside of Pythagoras since they have a zero Potential term V (2.6), which is attributed physical significance in [1] – see also [2] and [4].

Note *** Integer eigenvalues C=1, $\lambda_2 = 6$, $\lambda_3 = -7$

10 Records

This section gives the solution to (1.0), under conditions (1.1), for prime exponent, with the smallest k-value. There is no restriction on the unity roots being trivial.

10.1 n=3

This (19,21,52) solution has an exceptionally small k-value of 6, and is currently the record holder for the smallest k-value, for all odd, prime exponents, $n \geq 3$, where $z > y$ as per condition (1.1a), but is not of the 'x plus y' form, Section (5.3), which actually has a very low k-value, $k = 3$ for a cubic, arbitrary x and y.

n	x	y	z	k
3	19	21	52	6

n	x,y,z	P	Q	R	$\overline{P}$	$\overline{Q}$	$\overline{R}$
3	19,21,52	11	16	-9	-12	4	-29

n	x,y,z	α	β	γ	K	V
3	19,21,52	7	-3	-5	193	-192

In this solution, all x, y, z have factors of n of the $2mn+1$ form, for $n = 3$, they are simply constructed as follows:

19=6.3+1,

21=3.7, where $7 = 2.3 + 1$, (m=1)

52=4.13, where 13=4.3+1, (m=2).

For all $z > y$ solutions, Section (7), this is the smallest k-value yet seen, excluding the 'x plus y' solution – see the comment above. Lower k-values are obtained for z unconstrained, as given in Section (9.1), e.g. the (7,9,4) cubic exponent solution has $k = -4$.

All the unity roots are non-trivial, i.e. adhere to the definitions in Section (4.2).

10.2 n=5

The quintic with lowest k-value (2436) tabulated is (2,3,11) with a k-value of 2436, given table (7.4.1), but this is an 'extended form' of the 'x plus y' solution, see also Section (5.3.1) and so discounted.

The second lowest k-value (16695) for n=5, tabulated in Section (7.4.10), is for the solution (5,11,31), with the following, complete solution:

n	x	y	z	k
5	5	11	31	16695

n	x,y,z	P	Q	R	$\overline{P}$	$\overline{Q}$	$\overline{R}$
5	5,11,31	1 Note *	-51	-8	26 Note *	3	-4

n	x,y,z	α	β	γ	K	V
5	5,11,31	-5	14	-1	-95	96

Note * both P and $\overline{P}$ are trivial roots (4.0).

10.3 n=7

There are currently no known solutions for n=7 with k values less than 8 digits for x in the range 2 to 9999, y=x+1 to 9999. The listed solution (3,10,43) in table (7.5.2), has a 9 digit k-value but is is an ‘extended form’ of the ‘x plus y’ solution, see also Section (5.3.1), and hence considered invalid for record purposes.

10.4 n=11

There are currently no solutions for n=11 with k values less than 8 digits for x in the range 2 to 9999, y=x+1 to 9999. The solution (3,5,23) in table (7.6.2), has a 13 digit k-value but is is an ‘extended form’ of the ‘x plus y’ solution, see also Section (5.3.1), and hence considered invalid for record purposes.

10.5 n=13

There are currently no solutions for n=13 with k values less than 8 digits. For x in the range 2 to 9999, y=x+1 to 9999. The solution (3,5,53) in table (7.7.2) has a 20 digit k-value but is is an ‘extended form’ of the ‘x plus y’ solution, see also Section (5.3.1), and hence considered invalid for record purposes.

11 Unity Roots Table

Non-trivial roots only, Section (4.2).

Unity roots are supplied for x in the range 1 to 100, with a few larger x supplied that have numerous unity roots.

For each exponent, the unity root table has two columns, the first giving x, the second giving the unity roots ‘ q ’ satisfying the unity root definition

(11.1) $q^n \equiv 1 \bmod(x)$,

but where the non-triviality condition is imposed, i.e.

(11.2) $q \neq 1 \bmod(x)$: a non-trivial root,

so that the trivial value of unity is not-included. For odd exponent this means the unity root range is restricted to

(11.3) $1 < q < x-1$ odd exponent.

11.1 n = 3

x	non-trivial unity roots
7 (2.1.3+1)	2,4
9 (3.3)	4,7
13 (2.2.3+1)	3,9
14 (2.7)	9,11
18 (2.9) Note *	7,13
19 (2.3.3+1)	7,11
21 (3.7)	4,16
26 (2.13)	3,9
27 (3.9) Note *	10,19
28 (4.7)	9,25
31 (2.5.3+1)	5,25
35 (5.7)	11,16
36 (4.9) Note *	13,25
37 (2.6.3+1)	10,26
38 (2.19)	7,11
39 (3.13)	16,22
42 (6.7)	25,37
43 (2.7.3+1)	6,36
45 (5.9) Note *	16,31
49 (7.7) or (2.8.3+1)	18,30
52 (2.2.13)	9,29
54 (2.27) Note *	19,37

56 (8.7)	9,25
57 (3.19)	7,49
61 (2.10.3+1)	13,47
62 (2.31)	5,25
63 (9.7)	4,16,22,25,37,43,46,58
65 (5.13)	16,61
67 (2.11.3+1)	29,37
70 (10.7)	11,51
73 (2.12.3+1)	8,64
74 (2.37)	47,63
76 (4.19)	45,49
77 (11.7)	23,67
78 (6.13)	55,61
79 (2.13.3+1)	23,55
81 (3.27) Note *	28,55
84 (12.7)	25,37
86 (2.43)	49,79
90 (2.45) Note *	31,61
91 (2.15.3+1), (7.13)	9,16,22,29,53,74,79,81
93 (3.31)	25,67
97 (2.16.3+1)	35,61
98 (2.49)	67,79
99 (11.9) Note *	34,67
103 (2.17.3+1)	46,55
104 (8.13)	9,81
105 (15.7)	16,46
108 (3.3.3.4) Note *	37,73
109 (2.18.3+1)	45,63
111 (3.37)	10,100
112 (16.7)	65,81
114 (6.19)	7,49
117 (3.3.13)	16,22,40,55,61,79,94,100
119 (17.7)	18,86
120 to 188 ommitted	
189 (3.3.3.7)	37,46,64,100,109,127,163,172
190 to 272 ommitted	
273 (3.7.13)	16,22,79,100,172,211,235,256
274 to 1728 ommitted	
1729 (7.13.19)	144,172,191,235,438,562,638, 653,666,729,809,828,900,919, 932,989,1075,1166,1236,1303, 1394,1569,1626,1654,1660,17 17

Note *. These values of x are divisible by the exponent n, but do not have factors of the form $2mn+1$. In such a case, the roots are actually trivial, Section (4.1), for one or more of the exponent factors of x.

11.2 n =5

x	non-trivial unity roots
11 (2.1.5+1)	3,4,5,9
22 (2.11)	3,5,9,15
25 (5.5) Note *	6,11,16,21
31 (2.3.5+1)	2,4,8,16
33 (3.11)	4,16,25,31
41 (2.4.5+1)	10,16,18,37
44 (4.11)	5,9,25,37
50 (2.25) Note *	11,21,31,41
55 (5.11)	16,26,31,36
61 (2.6.5+1)	9,20,34,58
62 (2.31)	33,35,39,47
66 (6.11)	25,31,37,49
71 (2.7.5+1)	5,25,54,57
75 (3.25) Note *	16,31,46,61
77 (7.11)	15,36,64,71
82 (2.41)	37,51,57,59
88 (8.11)	9,25,49,81
93 (3.31)	4,16,64,70
99 (9.11)	37,64,82,91
100 (4.25) Note *	21,41,61,81
101 to 340 ommitted	
341	4,16,47,64,70,78,97,125,126,157,159,163,188,190 ,202,218,221,225,256,280,287,295,311,312

Note *. See comments for n=3, Section (11.1).

11.3 n =7

x	non-trivial unity roots
29 (2.2.7+1)	7,16,20,23,24,25
43 (2.3.7+1)	4,11,16,21,35,41
49 (7.7) Note *	8,15,22,29,36,43
58 (2.29)	7,23,25,45,49,53
71 (2.5.7+1)	20,30,32,37,45,48
86 (2.43)	11,21,35,41,47,59
87 (3.29)	7,16,25,49,52,82
98 (2.49) Note *	15,29,43,57,71,85
99, 100	none

Note *. See comments for n=3, Section (11.1).

11.4 n =11

x	non-trivial unity roots

23 (2.1.11+1)	2,3,4,6,8,9,12,13,16,18
46 (2.23)	3,9,13,25,27,29,31,35,39,41
67 (2.3.11+1)	9,14,15,22,24,25,40,59,62,64
69 (3.23)	4,13,16,25,31,49,52,55,58,64
89 (2.4.11+1)	2,4,8,16,32,39,45,64,67,78
92 (4.23)	9,13,25,229,4149,73,77,81,85
115 (5.23)	6,16,26,31,36,41,71,81,96,101
121 (11.11) Note *	12,23,34,45,56,67,78,89,100,111

Note *. See comments for n=3, Section (11.1).

11.5 n =13

x	non-trivial unity roots
53 (2.2.13+1)	10,13,15,16,24,28,36,42,44,46,47,49
79 (3.2.13+1)	8,10,18,21,22,38,46,52,62,64,65,67
106 (4.2.13+1)	13,15,47,49,63,69,77,81,89,95, 97,99
131 (5.2.13+1)	39,45,52,60,63,80,99,107,112,113
157 (6.2.13+1)	14,16,39,46,67,75,93,99,101,108,130,153
158 (2.79)	21,65,67,87,89,97,101,117,125,131,141,143
159 (3.53)	10,13,16,28,46,49,97,100,121,130,142,148
169 (13.13) Note *	14,27,40,53,66,79,92,105,118,131,144,157

Note *. See comments for n=3, Section (11.1).

11.6 n=17

x	non-trivial unity roots
103 (2.3.17+1)	8,9,13,14,23,30,34,61,64,66,72,76,79,81,93,100
137 (2.4.17+1)	16,34,38,50,56,59,60,72,73,74,88,115,119,122,123,133
206 (2.103)	9,13,23,61,79,81,93,111,117,133,137,167,169,175,179, 203
239 (2.7.17+1)	6,22,36,40,51,67,71,75,101,128,132,163,166,187,211,2 16
274 (2.137)	59,73,115,119,123,133,153,171,175,187,193,197,209,2 11,225,259
289 (13.13) Note *	18,35,52,69,86,103,120,137,154,171,188,205,222,239,2 56,273

Note *. See comments for n=3, Section (11.1).

11.7 n=19

x	non-trivial unity roots
191 (2.5.19+1)	5,6,25,30,32,36,55,69,107,121,125,136,150,153,154,160,177,180
229 (2.6.19+1)	16,17,27,42,43,44,53,57,60,61,104,121,161,165,203,214,218,225
361 (19.19)	20,39,58,77,96,115,134,153,172,191,210,229,248,267,286,305,324,343

11.8 n=23

x	non-trivial unity roots
47 (2.1.23+1)	2,3,4,6,7,8,9,12,14,16,17,18,21,24,25,27,28,32,34,36,37,42
94 (2.47)	3,7,9,17,21,25,27,37,49,51,53,55,59,61,63,65,71,75,79,81,83,89
139 (2.3.23+1)	6,34,36,,44,45,52,55,57,63,64,65,77,79,80,91,100,106,112,116,125,129,131
141 (3.47)	4,7,16,25,28,34,37,49,55,61,64,79,97,100,103,106,112,115,118,121,130,136
188 (4.47)	9,17,21,25,37,49,53,61,65,81,89,97,101,121,145,149,153,157,165,169,173,177
235 (4.47)	6,16,21,36,51,56,61,71,81,96,101,106,111,121,126,131,136,166,191,196,206,216
277 (2.6.23+1)	16,19,27,30,52,69,84,131,155,157,164,169,175,201,203,211,213,218,236,256,264,273
278 (2.139)	45,55,57,63,65,77,79,91,125,129,131,145,173,175,183,191,203,219,239,245,251,255
282 (6.47)	7,25,37,49,55,61,79,97,103,115,121,145,157,169,175,205,241,247,253,259,271,277
329 (7.47)	8,36,50,64,71,106,148,155,162,169,183,190,197,204,225,239,253,260,267,288309,316
376 (8.47)	9,17,25,49,65,81,8,97,121,145,153,169,177,209,22,5241,249,289,337,345,353,361
417	34,52,55,64,79,91,100,106,112,145,175,184,196,202,268,322,343,355,358,394,403,409
423	28,37,55,64,100,118,136,145,190,244,253,262,271,289,298,307,316,343,361,379,388,397
461	14,23,30,33,68,153,167,181,196,229,262,292,298,322,348,359,400,416,420,439,440,441
470	21,51,61,71,81,101,111,121,131,191,241,251,271,291,331,341,361,371,401,431,441,451
517	12,34,56,89,100,111,122,144,155,166,177,243,298,309,331,353,397,408,430,441,474,507

529	24,47,70,93,116,139,162,185,208,231,254,277,300,323,346,369,392,415,438,461,484,507

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http://www.fast-print.net/bookshop/823/unity-root-matrix-theory-physics-in-integers

This book is broken into six separate papers, each paper is given a specific reference #1 to #6 as follows:

[1]#1 Unity Root Matrix Theory Foundations
[1]#2 see [5], below
[1]#3 Geometric and Physical Aspects
[1]#4 Solving Unity Root Matrix Theory
[1]#5 Unifying Concepts
[1]#6 A Non-unity Eigenvalue

[2] Unity Root Matrix Theory Overview, R J Miller, a free PDF document available for download:

http://www.urmt.org/presentation_URMT_shortform.pdf

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http://wwww.urmt.org/sections1to7_18122004.pdf

[4] The Modified FLT Equation (MFLT).

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[5] Pythagorean Triples as Eigenvectors and Related Invariants, R. J. Miller, 2010, a free PDF document available for download:

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This paper is equivalent to Paper 2 in [1], i.e. Ref. [1]#2.

[6] Unity Root Matrix Theory and the Riemann Hypothesis, R. J. Miller, 2013, a free PDF document available for download:

http://www.urmt.org/urmt_riemann_link.pdf

[7] I.Niven, S.Zuckerman, H.L.Montgomery, An Introduction to the Theory of Numbers, 5th Edition, John Wiley & Sons, Inc. 1991. ISBN 0-471-54600-3.

[8] H Davenport, The Higher Arithmetic, 6th Edition, Cambridge University Press, ISBN 0-521-42227-2.

[9] Unity Root Matrix Theory web site, http://www.urmt.org.

Example reference syntax used throughout:

[1]#3,8 : Book 1, Paper #3, Section (8)
(7.4) : this document, sub-section (7.4)
(2.0) : this document, equation (2.0)

As regards existing, published work in the field of Unity Root Matrix Theory, since this is a relatively new subject area (less than ten years old) then, to the author's best knowledge, the only currently available texts are those referenced from the web site [9], which the reader may wish to visit occasionally since new, free material is added every few months.